MA 842: Explicit methods for elliptic and hyperelliptic curves
Spring 2017
Problem Set 3
Due: February 20, 2017

Let $E/\mathbb{Q}$ be an elliptic curve.

(1) Let E_p denote the reduction of E modulo a prime p. Prove that in the cases when $p \mid \Delta_E$, we still have $a_p = p + 1 - \#E_p(\mathbb{F}_p)$.
(2) Prove that any elliptic curve $E'/\mathbb{Q}$ that is (rationally) isogenous to E has the same analytic rank.
(3) Read about isogeny invariance of the BSD quotient for E (describing the leading coefficient of the L-function) and sketch a proof.
(4) We've spent some time discussing $L^*(E, 1)$, the leading coefficient of the Taylor series expansion of the L-function associated to E at $s = 1$. Are there conjectures on other special values of the L-function associated to E? Read and summarize what you find.

Let $A : y^2 + xy = x^3 - x^2 - 79x + 289$.

(1) Compute the Mordell-Weil group $A(\mathbb{Q})$. (Feel free to use Sage/Magma.)
(2) Use Dokchitser's L-function calculator to compute an approximation to the L-function of A. Compute all of the quantities appearing in the BSD quotient for $A/\mathbb{Q}$ and explain which computations are/can be made rigorous.
(3) Pick a prime p of good ordinary reduction for A. Use Sage to compute the p-adic L-function attached to A, and compute all of the quantities appearing in the p-adic BSD quotient for $A/\mathbb{Q}$. Explain which computations are/can be made rigorous.