MA 842: Explicit methods for elliptic and hyperelliptic curves
Spring 2017
Problem Set 6
Due: April 19, 2017

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(1) Let $E : y^2 = x^3 + x + 1$ be an elliptic curve over $\mathbb{Q}$. For each prime p of good reduction, consider
$$a_p = p + 1 - \#E_p(F_p),$$
the trace of Frobenius. Consider the normalized trace $a_p/\sqrt{p}$. Use Sage to plot the normalized traces for $p < N$. (You can pick how large you take N. See how far you can go!).

(2) Let $X : y^2 = x^5 - x + 1$ be a hyperelliptic curve over $\mathbb{Q}$. For each prime p of good reduction, let $L_p(T) = \prod_{i=1}^{4}(1 - \alpha_i T)$ denote the numerator of the zeta function of $X_p/\mathbb{F}_p$, and consider the normalized L-polynomial $\overline{L}_p(T) = L_p(T/\sqrt{p})$. The polynomial $\overline{L}_p(T)$ has the form
$$\overline{L}_p(T) = T^4 + a_1 T^3 + a_2 T^2 + a_1 T + 1.$$
   (a) Fix an upper bound N (in the realm of interesting but not unreasonable).
   (b) Use Sage to plot the distribution of a_1 for $p < N$.
   (c) Use Sage to plot the distribution of a_2 for $p < N$.

(3) Do some reading for your final[1] project. Submit the following:
   (a) a working title,
   (b) a paragraph describing what you will do, and
   (c) two citations of sources you have consulted.

[1] Final projects will be due at 12pm on May 8.