

**Table 10.4** Special Series and Convergence Tests

Series or Test	Form of Series	Condition for Convergence	Condition for Divergence	Comments
Geometric series	$\sum_{k=0}^{\infty} ar^k, a \neq 0$	$ r  < 1$	$ r  \geq 1$	If $ r  < 1$ , then $\sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$ .
Divergence Test	$\sum_{k=1}^{\infty} a_k$	Does not apply	$\lim_{k \rightarrow \infty} a_k \neq 0$	Cannot be used to prove convergence
Integral Test	$\sum_{k=1}^{\infty} a_k$ , where $a_k = f(k)$ and $f$ is continuous, positive, and decreasing	$\int_1^{\infty} f(x) dx$ converges.	$\int_1^{\infty} f(x) dx$ diverges.	The value of the integral is not the value of the series.
$p$ -series	$\sum_{k=1}^{\infty} \frac{1}{k^p}$	$p > 1$	$p \leq 1$	Useful for comparison tests
Ratio Test	$\sum_{k=1}^{\infty} a_k$	$\lim_{k \rightarrow \infty} \left  \frac{a_{k+1}}{a_k} \right  < 1$	$\lim_{k \rightarrow \infty} \left  \frac{a_{k+1}}{a_k} \right  > 1$	Inconclusive if $\lim_{k \rightarrow \infty} \left  \frac{a_{k+1}}{a_k} \right  = 1$
Root Test	$\sum_{k=1}^{\infty} a_k$	$\lim_{k \rightarrow \infty} \sqrt[k]{ a_k } < 1$	$\lim_{k \rightarrow \infty} \sqrt[k]{ a_k } > 1$	Inconclusive if $\lim_{k \rightarrow \infty} \sqrt[k]{ a_k } = 1$
Comparison Test	$\sum_{k=1}^{\infty} a_k$ , where $a_k > 0$	$a_k \leq b_k$ and $\sum_{k=1}^{\infty} b_k$ converges.	$b_k \leq a_k$ and $\sum_{k=1}^{\infty} b_k$ diverges.	$\sum_{k=1}^{\infty} a_k$ is given; you supply $\sum_{k=1}^{\infty} b_k$ .
Limit Comparison Test	$\sum_{k=1}^{\infty} a_k$ , where $a_k > 0, b_k > 0$	$0 \leq \lim_{k \rightarrow \infty} \frac{a_k}{b_k} < \infty$ and $\sum_{k=1}^{\infty} b_k$ converges.	$\lim_{k \rightarrow \infty} \frac{a_k}{b_k} > 0$ and $\sum_{k=1}^{\infty} b_k$ diverges.	$\sum_{k=1}^{\infty} a_k$ is given; you supply $\sum_{k=1}^{\infty} b_k$ .
Alternating Series Test	$\sum_{k=1}^{\infty} (-1)^k a_k$ , where $a_k > 0$	$\lim_{k \rightarrow \infty} a_k = 0$ and $0 < a_{k+1} \leq a_k$	$\lim_{k \rightarrow \infty} a_k \neq 0$	Remainder $R_n$ satisfies $ R_n  \leq a_{n+1}$
Absolute Convergence	$\sum_{k=1}^{\infty} a_k$ , $a_k$ arbitrary	$\sum_{k=1}^{\infty}  a_k $ converges.		Applies to arbitrary series