MA 225 PRACTICE MIDTERM II

1. (10 points) Let D be the region in space bounded by the surface $x^2 + y^2 = 9$ and the planes z = 0 and z = 1. Find $\iiint_D z^2 dV$. Sketch D (3 pts), set up an appropriate iterated integral (4 pts), and then solve the integral (3 pts). 2. (10 points) Let $f(x, y) = x^4 + 2y^2 - 4xy$. Find all critical points of f(x, y) (5 pts), and determine whether they are local maxima, minima, or saddle points (5 pts).

3. (10 points) The iterated integral

$$\int_0^{\sqrt{\pi}} \int_x^{\sqrt{\pi}} \sin(y^2) \, dy \, dx$$

can't be evaluated directly. Sketch the region of integration (3 pts). Then reverse the order of integration to get a new iterated integral (4 pts). Evaluate the integral (3 pts).

4. (10 points) Let D be the hemisphere defined by $z \ge 0$ and $x^2 + y^2 + z^2 \le 4$. Assume constant density throughout D. The center of mass of D is on the z-axis, so it must be a point with coordinates (0, 0, c). Find c. (Hint: I did this problem using spherical coordinates, and then I did it using cylindrical coordinates. They both seem to work fine. But don't use rectangular coordinates!)

5. (10 points) Let P be the parallelogram in the plane with vertices (0,0), (1,0), (1,1), and (2,1). Calculate $\iint_P y \, dA$ using the substitution x = u + v, y = v.