MA 225 PRACTICE FINAL

1. (12 points) Answer the following questions about 3D vector geometry.

   (a) (3 pts) Find a vector which is normal to the plane $2x - 3z = 1$.

   (b) (3 pts) Find the angle between the vectors $\langle 1, 0, 1 \rangle$ and $\langle -1, 1, 0 \rangle$.

   (c) (3 pts) If $\mathbf{v}$ and $\mathbf{w}$ are any vectors in space, then $\mathbf{v} \cdot (\mathbf{v} \times \mathbf{w})$ equals:

   (d) (3 pts) Write down the interpretation of $|\mathbf{v} \times \mathbf{w}|$ as an area.

2. (6 points) Let $\mathbf{F} = \langle f, g \rangle$ be a vector field with continuous partial derivatives on a simply-connected region R in the plane. Which of the following statements does not mean the same as the others?

(1) The divergence of $\mathbf{F}$ is 0 everywhere on R.
(2) $\partial g/\partial x - \partial f/\partial y = 0$ everywhere on R.
(3) $\oint_C \mathbf{F} \cdot d\mathbf{r} = 0$ for every closed curve C in R.
(4) $\mathbf{F} = \nabla \phi$ for a function ϕ defined on R.

3. (6 points) Suppose that x, y and z are positive numbers satisfying $x + 2y + z = 12$. Find the largest possible value of xyz.

4. (8 points) Let $\mathbf{F} = \langle xy, x+y \rangle$. Let C be the triangle in the plane with vertices $(0,0)$, $(1,0)$ and $(1,1)$, oriented counterclockwise. Find $\oint_C \mathbf{F} \cdot d\mathbf{r}$.

5. (6 points) Find the Jacobian of the transformation $x = ve^u$, $y = ve^{-u}$.

6. (10 points) Let $z = f(x, y)$ be a differentiable function. Let r and θ be the usual variables from polar coordinates, so that $x = r\cos\theta$ and $y = r\sin\theta$. At the point $(r, \theta) = (1, \pi/4)$, find $\partial z/\partial r$ and $\partial z/\partial\theta$ in terms of $\partial z/\partial x$ and $\partial z/\partial y$.

7. (6 points) Sketch the graph of $z = \sqrt{x^2 + y^2}$.

8. (10 points) Let S be the filled-in square in the xz-plane with vertices $(0,0,0)$, $(1,0,0)$, $(1,0,1)$ and $(0,0,1)$. Orient S in the positive y-direction. Let $\mathbf{F}$ be the vector field $\langle -y, z, x\rangle$. Find the flux of $\mathbf{F}$ through S.

9. (10 points) Find $\iiint_D z^2 dV$, where D is the region in space defined by the inequalities $x^2 + y^2 \leq 4$, $0 \leq z \leq 1$.

10. (10 points) Let $\phi(x, y, z) = x^3y + z$. Find $\int_C \nabla\phi \cdot d\mathbf{r}$, where C is any curve starting at $(1, 0, 0)$ and ending at $(1, 1, 1)$.