MA 341 HW #10, due Friday, Apr. 14

1. Find all solutions to the Diophantine equation $x^2 + y^2 = 65$, where x and y are positive integers.

2. Draw a picture of the Gaussian integers $\mathbf{Z}[i]$, and circle multiples of $1 + 2i$ until you see a pattern.

3. Find a complete set of representatives for $\mathbf{Z}[i]$ modulo $1 + 2i$.

4. Find Gaussian integers $q, r \in \mathbf{Z}[i]$ such that $4+i = (1+2i)q+r$, where the remainder r satisfies $N(r) < N(1 + 2i) = 5$. (You can use the picture you drew above to do this.)

5. Factor the following Gaussian integers into primes in $\mathbf{Z}[i]$: 3, 5, 65, $3 + 5i$.

6. Let d be the gcd of $-1+3i$ and $-4+7i$. Find d. Can you find Gaussian integers x, y so that $(-1 + 3i)x + (-4 + 7i)y = d$?