# MA 341 HW #12, due Friday, Apr. 28

1. Factor $9 + 7i$ into Gaussian primes.
2. Find an inverse to $1 + i$ modulo 7.
3. Let $p_1, p_2, p_3$ be distinct primes which are each congruent to 1 modulo 4. How many solutions are there to $x^2 + y^2 = p_1 p_2 p_3$ for positive integers $x, y$?
4. Let $a, b \in \mathbf{Z}$ be relatively prime, and let $\alpha = a + bi$. Show that the gcd between $\alpha$ and $\overline{\alpha}$ is either 1 or $1 + i$.
5. Let $p \equiv 3 \pmod 4$ be prime. For $a, b \in \mathbf{Z}$, show that $(a + bi)^p \equiv a - bi \pmod p$. (Remember that $p$ divides the binomial coefficients $\binom{p}{a}$ for $1 \leq a \leq p - 1$.)
6. Continuing with the previous problem, suppose $p \equiv 3 \pmod 4$, and suppose $\alpha \in \mathbf{Z}[i]$ is not divisible by $p$. Show that $\alpha^{p^2-1} \equiv 1 \pmod p$.