

## MA 341 Midterm, due Friday, Mar. 24

1. Let  $n$  be the number whose base 10 expansion is  $100\cdots 001$ , where there are 888 zeros. Is  $n$  prime or composite?
2. Compute  $3^{3^{101}} \pmod{17}$ .
3. When the fraction  $1/17$  is written to base 2 (binary), what is the period? Are there any other integers  $n \geq 1$  such that  $1/n$  has the same period?
4. Suppose  $b$  is a primitive root modulo a prime  $p$ . For an integer  $n$ , find a formula for the order of  $b^n$  modulo  $p$ . Prove your answer.
5. Find integers  $x, y, z$  such that  $55x + 35y + 77z = 1$ . Be sure to show your method.
6. Suppose  $b$  is a primitive root modulo a prime  $p$ , and suppose that  $p \equiv 1 \pmod{4}$ . Let  $c = b^{(p-1)/4}$ . Show that  $c^2 \equiv -1 \pmod{p}$ .
7. Describe the *complete* set of solutions to the simultaneous congruences:

$$x \equiv 26 \pmod{100}$$

$$x \equiv 36 \pmod{90}$$