MA541 Midterm

Due Oct. 23

Your solutions to each of the following problems will be graded by this method: I will pretend not to have the slightest idea of how to prove the assertion, and then see how well you can convince me that the assertion is true. Therefore I will be unforgiving if your proof is confusing, vague, or uses variables in an imprecise way, even if I can tell you have the basic idea of the proof. Take special care in using the right quantifiers ("for every", "there exists", etc.). Also keep in mind that every proof technique you will need was covered at some point in the lectures. You may consult your lecture notes and the textbook, but not other books, internet sources, or any one else.

1. Let G and G' be groups, and let $f\colon G \to G'$ be a homomorphism. This means that $f(gh) = f(g)f(h)$ for all $g, h \in G$. Let K be the subset of elements of G which map to the identity of G' under f. Show that K is a subgroup of G.

2. Let g be an element of a group G. Show that the map $f\colon G \to G$ defined by $f(x) = g^{-1}xg$ is an automorphism (meaning an isomorphism of G with itself).

3. Let g and h be elements in a group. Show that gh has the same order as hg. (Don't neglect the case that the order of these elements could be infinite.)

4. Let G be a group with at least two elements which has no subgroups other than itself and the trivial subgroup. Show that G is a cyclic group whose order is a prime number.

5. Let H be a subgroup of the symmetric group S_n for some $n \geq 1$. Show that either all the elements of H are even, or else exactly half of the elements of H are even.

6. Let g and h be elements of an abelian group. Let m be the order of g and let n be the order of h. Assume that m and n are relatively prime. Show that $\langle g, h\rangle = \langle gh\rangle$.