MA541 Midterm

Due Oct. 23

For this exam, you may consult your lecture notes and the textbook, but not other books, internet sources, or anyone else.

1. Let g be an element of a group G. Show that the map $f\colon G \to G$ defined by $f(x) = g^{-1}xg$ is an automorphism (meaning an isomorphism of G with itself).

2. Let g be an element of a group G, and let $H = \{x \in G | xg = gx\}$. Show that H is a subgroup of G.

3. Recall that if g is an element of a group, the *order* of g is the cardinality of $\langle g \rangle$. Let g and h be elements in a group. Show that gh has the same order as hg. (Don't neglect the case that the order of these elements could be infinite.)

4. Does S_5 have any elements of order 6? If so, how many?

5. Suppose a group has at least two elements, and that it has no proper subgroups other than itself and the trivial subgroup. Show that this group is a cyclic group whose order is a prime number.

6. Consider the group $\mathbf{R}^*$ of nonzero real numbers under multiplication. Find a subgroup $H \leq \mathbf{R}^*$ such that $(\mathbf{R}^* : H) = 2$.