MA 843 Assignment 4, due Tues. 10/25

1. Let χ be a Dirichlet character modulo N, which we view simultaneously as a character of $(\mathbf{A}_{\mathbb{Q}}^{\text{fin}})^\times/\mathbb{Q}_{>0}^\times = \hat{\mathbb{Z}}^\times$. In lecture we described a map $f \mapsto \phi_f$ from $M_k(\Gamma_1(N))$ into M_k. Show that the inverse image of $M_k(\chi)$ is $M_k(N,\chi)$.

2. Let V be a smooth representation of an abelian profinite group Z. Show that V is the direct sum of its χ-isotypic subspaces $V(\chi)$, as χ runs through characters of Z. (In particular M_k is the direct sum of the $M_k(\chi)$.)

3. Let G be a locally profinite group, and let $\rho\colon G \to \mathrm{GL}(V)$ be a smooth representation. Show that the following definitions of admissibility are equivalent:

    (a) V^K is finite-dimensional for each open compact subgroup $K \subset G$.

    (b) The "evaluation map" $V \to \check{\check{V}}$ is an isomorphism.

4. Let F be a p-adic field, let $G = \mathrm{GL}_n(F)$, and let B be the Borel subgroup of upper triangular matrices. Describe the module δ_B.