

## First midterm Review

1.  $g'(x) = \cos(x)$
2.  $h'(x) = 2x\sqrt{1+x^6}$
3.  $f'(x) = -e^x \sin^3(e^x)$
4.  $g'(x) = \sin x(1 + \cos^2 x)^{10} + \cos x(1 + \sin^2 x)^{10}$
5.  $f(x) = \int_1^x \frac{2^t}{t} dt$
6. (a)  $f(t)$  is the rate (or derivative) of the loss in value function. So, by the fundamental theorem of calculus,  $\int_0^t f(s) ds$  represents the value lost at time  $t$ .  
(b)  $C(t)$  represents the average cost of overhauling the machine at time  $t$  (the cost of the overhaul plus the loss in value). A minimum  $T$  for  $C(t)$  is the best time for the company to overhaul the machine.  
(c)  $C(t)$  has a local minimum or maximum when  $C'(t) = 0$  or undefined.

$$C'(t) = -\frac{1}{t^2} \left[ A + \int_0^t f(s) ds \right] + \frac{1}{t} f(t).$$

$C'(T) = 0$  when  $\frac{1}{T} = \frac{1}{T^2} \left[ A + \int_0^T f(s) ds \right]$ . You can assume that  $T \neq 0$  (why?), so

$$f(T) = \frac{1}{T} \left[ A + \int_0^T f(s) ds \right] = C(T)$$

is the only place where a local minimum can occur.

7.  $-\sin(\cos x) + C$

8.  $\frac{1}{2} \sin^{-1}(x^2) + C$
9.  $x \tan^{-1} x - \frac{1}{2} \ln(1 + x^2) + C$
10.  $\frac{1}{3}(x^2 + 1)^{3/2} - (x^2 + 1)^{1/2} + C$
11.  $\frac{1}{37}(3e^{t/3} \sin(2t) - 18e^{t/3} \cos(2t)) + C$
12.  $-\frac{1}{\pi} \sin(\pi/x) + C$
13.  $-\tan^{-1}(\cos x) + C$
14.  $\frac{2}{5}x^{5/2} \ln x - \frac{4}{25}x^{5/2} + C$
15.  $\frac{1}{2}x^2 e^{x^2} - \frac{1}{2}e^{x^2} + C$
16.  $\frac{1}{7} \sin^7 x - \frac{1}{9} \sin^9 x + C$
17.  $\frac{1}{3} \tan^3 \theta - \tan \theta + \theta + C$
18.  $\frac{1}{14} \sec^7(2x) - \frac{1}{10} \sec^5(2x) + C$
19.  $\frac{1}{5}(9 - x^2)^{5/2} - 3(9 - x^2)^{3/2} + C$
20.  $\frac{1}{5}(t^2 + 2)^{5/2} - \frac{4}{3}(t^2 + 2)^{3/2} - 4(t^2 + 2)^{1/2} + C$
21.  $\frac{1}{16} \frac{\sqrt{x^2-16}}{x} + C$
22.  $\ln \left| \frac{\sqrt{t^2-6t+13}}{2} + \frac{t-3}{2} \right| + C$
23.  $6 \sin^{-1}\left(\frac{x-2}{2}\right) - \frac{(x-2)\sqrt{4x-x^2}}{2} - 4\sqrt{4x-x^2} + C$
24.  $\frac{1}{10}$
25.  $-\frac{1}{3} \ln(4)$
26.  $\frac{1}{3}a^3$
27.  $\frac{16}{3} \ln(4) - \frac{28}{9}$
28.  $\frac{1}{2}(e^t - \sin t - \cos t)$
29.  $\frac{4}{3} \ln(2) + \frac{2}{3} \ln(5) - \frac{2}{3} \ln(4)$  (This actually equals  $\frac{2}{3} \ln(5)$ , but you don't have to simplify it if you don't want to.)

30.  $\ln(\sqrt{2} + 1)$

31.  $2 - 2e^{-T} - 2Te^{-T} - T^2e^{-T}$

32. 2

33. The answers all differ by a constant (use a double angle formula to see this!)