

First midterm Review

Find the derivative of the function.

1. $g(x) = \int_1^x \cos(t) dt$

2. $h(x) = \int_0^{x^2} \sqrt{1+r^3} dr$

3. $f(x) = \int_{e^x}^0 \sin^3(t) dt$

4. $g(x) = \int_{\sin x}^{\cos x} (1+v^2)^{10} dv$

5. Find a function f such that $f(1) = 0$ and $f'(x) = \frac{2^x}{x}$.

6. A manufacturing company owns a major piece of equipment that depreciates at the (continuous) rate $f = f(t)$, where t is the time measure in months since its last overhaul. Because a fixed cost A is incurred each time the machine is overhauled, the company wants to determine the optimal time T (in months) between overhauls.

(a) Explain why $\int_0^t f(s) ds$ represents the loss in value of the machine over a period of time t since the last overhaul.

(b) Let $C = C(t)$ be given by:

$$C(t) = \frac{1}{t} \left[A + \int_0^t f(s) ds \right].$$

What does C represent and why would the company want to minimize C ?

(c) Show that C has a minimum value at the numbers $t = T$ where $C(T) = f(T)$.

Evaluate the indefinite integral.

7. $\int \sin x \cos(\cos x) dx$

8. $\int \frac{x}{\sqrt{1-x^4}} dx$

9. $\int \tan^{-1} x dx$

10. $\int \frac{x^3}{\sqrt{x^2+1}} dx$

11. $\int e^{t/3} \sin(2t) dt$

12. $\int \frac{\cos(\pi/x)}{x^2} dx$

13. $\int \frac{\sin x}{1+\cos^2 x} dx$

14. $\int x^{3/2} \ln x dx$

15. $\int x^3 e^{x^2} dx$

16. $\int \sin^6 x \cos^3 x dx$

17. $\int \tan^4 \theta d\theta$

18. $\int \tan^3(2x) \sec^5(2x) dx$

19. $\int x^3 \sqrt{9-x^2} dx$

20. $\int \frac{t^5}{\sqrt{t^2+2}} dt$

21. $\int \frac{dx}{x^2 \sqrt{x^2-16}}$

22. $\int \frac{dt}{\sqrt{t^2-6t+13}}$

23. $\int \frac{x^2}{\sqrt{4x-x^2}} dx$

Evaluate the definite integral.

24. $\int_0^1 (1-x)^9 dx$

25. $\int_1^2 \frac{1}{2-3x} dx$

26. $\int_0^a x\sqrt{a^2-x^2} dx$

27. $\int_1^4 \sqrt{t} \ln t dt$

28. $\int_0^t e^s \sin(t-s) ds$

29. $\int_7^{14} \frac{1}{x-\sqrt{x+2}} dx$

30. $\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}} dt$

31. A particle that moves along a straight line has velocity $v(t) = t^2 e^{-t}$ meters per second after t seconds. How far will it travel in T seconds?

32. Suppose that $f(1) = 2$, $f(4) = 7$, $f'(1) = 5$, $f'(4) = 3$, and f'' is continuous. Find the value of $\int_1^4 x f''(x) dx$.

33. Evaluate $\int \sin x \cos x dx$ using four different methods:

- (a) the substitution $u = \cos x$;
- (b) the substitution $u = \sin x$;
- (c) the identity $\sin(2x) = 2 \sin x \cos x$;
- (d) integration by parts.

Explain the different appearances of the answers.