Math 123, Practice Exam #2, November 8, 2000

1. Find the following:

   (a)
   $$\lim_{y \to \infty} \left(1 + \frac{2}{y}\right)^y$$

   (b)
   $$\lim_{x \to 0} \frac{\cos\left(\sqrt{5}\, x\right) - 1}{x^2}$$

   (c)
   $$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$$

   (d) The horizontal and vertical asymptotes of
   $$y = \frac{4 - 3x}{\sqrt{16x^2 + 1}}$$

   (e) $f'(x)$ where
   $$f(x) = \frac{\ln x}{x}$$

   (f) $f'(x)$ where
   $$f(x) = x^{x^2}$$

   (g) $f'(x)$ where
   $$f(x) = \arctan(x^3)$$

2. Find the equation for the tangent line to the curve given by the equation $y^2 = \cos(xy) + x$ through the point $(x, y) = (0, -1)$.

3. A man 6 feet tall is walking away from a light pole which is 30 feet high. If the tip of his shadow is moving at a rate equal to the distance between him and the light pole (in feet) then how fast is the man walking when he is 24 feet from the pole?

4. A spherical snowball is melting at a rate equal to its surface area. How fast is its radius shrinking when its volume is equal to its surface area?

5. Two nonnegative numbers are such that the sum of the first number and 3 times the second number equals 10. Find these numbers if the sum of their squares is as small as possible.

6. Consider the functon
   $$f(x) = 3x^4 - 4x^3 + 20000$$

(a) On what interval(s) is f increasing?

(b) On what interval(s) is f concave down?

(c) Find the inflection point(s) of f.

(d) Find the critical points of f.

(e) Find the local maximum (maxima) of f.

(f) Find the global minimum of f on the interval $[-2, 3]$.

7. Suppose the graph on the following page is of $y = f'(x)$ (**NOT** $f(x)$).

(a) Find the critical numbers of f.

(b) On what interval(s) is f increasing?

(c) On what interval(s) is f concave down?

(d) Find the values of x on the interval $(-\infty, \infty)$ where f has a local minimum.

(e) Find the values of x on the interval $[0, 4]$ where f has a global minimum.

(f) Find the x values of all inflection points of f.