Math 563A1, Final Exam, December 8, 2000
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The exam is open book and you may work with others although you must write up the solutions yourself. Good luck!

1. (10 points) A surface with parametrization

$$x(u, v) = (\cos u \cos v, \sin u \cos v, v + \sin u)$$

   is called The Pretzel Surface. Find it principal curvatures and principal directions. Also, find its Gaussian and Mean Curvatures.

2. (10 points) Exercises 5.5.5 and 5.5.7 from the text.

3. (10 points) Find the lines of curvatures and the asymptotic curves of $z = xy$. See Exercise 2.4.2 for the definition of lines of curvature and Exercise 4.2.3 for the definition of asymptotic curves.

4. (10 points) Let $f : \mathbf{R} \to \mathbf{R}$ where $f(u) > 0$. Furthermore, suppose the the resulting surface of revolution around the first axis (in $\mathbf{R}^3$) has constant curvature -1. Find the equation that f must satisfy.

5. (10 points) Exercises 6.2.1 and 6.3.1, 6.3.2 from the text.