Math 563A1, Final Exam, December 3, 2001
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The exam is open book and you may work with others although you must write up the solutions yourself. Good luck!

1. (10 points) Consider the map $\alpha : \mathbf{R} \to \mathbf{R}^3$ given by

$$\alpha(t) := \begin{cases} (t^2, e^{-\frac{1}{t^2}}, 2), & \text{if } t \neq 0; \\ (0,0,2) & \text{if } t = 0 \end{cases}.$$

   (a) For what values of t is $\alpha(t)$ a differentiable curve?

   (b) For what values of t is $\alpha(t)$ a regular curve?

   (c) Calculate its Frenet frame, curvature and torsion.

2. (20 points) A surface with parametrization

$$x(u,v) = (\cos u \cos v, \sin u \cos v, v + \sin u)$$

   is called the Pretzel Surface. Find its principal curvatures and principal directions. Also, find its Gaussian and Mean Curvatures.

3. (15 points) Let Σ be a regular surface and let $\alpha : (a,b) \to \Sigma$ be a unit speed curve. Consider the parametrized surface $\widetilde{\Sigma}$ with the chart $x : (a,b) \times (-\epsilon, \epsilon) \to \widetilde{\Sigma}$ given by

$$x(s,v) = \alpha(s) + vB(s)$$

   where $B(s)$ is the binormal vector of $\alpha(s)$ and where ϵ is a small positive number. Prove that if ϵ is small then $\widetilde{\Sigma}$ is a regular surface over which α is a geodesic.

4. (15 points) Consider the surface Σ given by the chart $x(u,v) = (u, v, u^2 - v^2)$. Find the asymptotic curves on Σ.

5. (10 points) Let Σ be a regular, compact, orientable surface embedded in $\mathbf{R}^3$ which is not homeomorphic to S^2. Prove that there are points on Σ where the Gaussian curvature is positive, negative, and zero.

6. (30 points) Exercises 6.3.2, 3; Exercises 6.5.2; Exercises 6.6.4, 6