

Math 123, Practice Exam Questions for Exam #1, October 10, 2003

Unless explicitly stated, full credit will be awarded only if the solution is correct and the steps leading up to it are correct.

1. Calculate the following:

(a)

$$\lim_{x \rightarrow -1} \frac{x^2 - 4x - 5}{x + 1}$$

(b)

$$\lim_{x \rightarrow 3} \frac{5x^2}{2x - 1}$$

(c)

$$\lim_{x \rightarrow 2^+} \frac{4 - x^2}{|2 - x|}$$

(d)

$$\lim_{x \rightarrow 1} \sqrt{\frac{2x^3 - 3x + 5}{2 - x}}$$

(e)

$$\lim_{x \rightarrow 3} f(x) \quad \text{where} \quad f(x) = \begin{cases} x^2 & \text{if } x > 3 \\ 8 & \text{if } x = 3 \\ 12 - x & \text{if } x < 3 \end{cases}$$

(f)

$$\lim_{x \rightarrow 3^-} \frac{x + 3}{x^2 - 9}$$

(g) The horizontal and vertical asymptotes of

$$y = \frac{4 - 3x}{\sqrt{16x^2 + 1}}$$

(h) $f'(x)$ where

$$f(x) = \sin(x^{100})$$

(i) $f'(x)$ where

$$f(x) = \sqrt{e^{2x} + 7x}$$

(j) $f'(x)$ where

$$f(x) = 10^{\cos x}$$

(k) $f'(x)$ where

$$f(x) = \frac{\ln x}{x}$$

(l) $f'(x)$ where

$$f(x) = x^{x^2}$$

(m) $f'(x)$ where

$$f(x) = \arctan(x^3)$$

2. Consider the function

$$f(x) = \begin{cases} x - c, & \text{if } x > 2; \\ 3x^2, & \text{if } x \leq 2 \end{cases}$$

where c is a real number.

(a) What value of c makes the function f continuous everywhere?

(b) Calculate $f'(7)$

(c) Calculate $f'(-1)$

(d) Calculate $f'(2)$

3. Compute the derivative of the following functions:

(a)

$$f(x) = \pi^4$$

(b)

$$f(x) = 3x^5 - x^2 + 9$$

(c)

$$f(x) = \frac{2}{x^2} - 3\sqrt{x}$$

(d)

$$f(x) = \frac{x^2}{2x - 3}$$

(e)

$$f(x) = x^2 e^x$$

(f)

$$f(t) = t \sin t$$

4. Find the equation for the tangent line to the curve $y = f(x)$ where through the point $(1, -3)$ where

$$f(x) = x^8 - 4x.$$

5. Suppose an object is moving along the real line with its position at time t given by the function $s(t) = \frac{1}{3}t^3 - 3t^2 - 7t + 10$.

- (a) When is the object at rest?
- (b) When is the object decelerating?
- (c) Find the velocity of the object at time $t = 2$.

6. Find the equation for the tangent line to the curve given by the equation $\cos(xy) - 3y^3 = e^x + 1$ through the point $(0, -1)$.

7. Consider the graph of $y = f(x)$ on the next page.

- (a) Where is f undefined?
- (b) Where is f not continuous?
- (c) Where is f not differentiable?
- (d) On what interval(s) is f' positive? Where does f' vanish?
- (e) On what interval(s) is f concave down?
- (f) What are
 - i.

$$f'(6)$$

ii.

$$\lim_{x \rightarrow -5^+} f(x)$$

iii.

$$\lim_{x \rightarrow -1} f(x)$$

iv.

$$\lim_{x \rightarrow -\infty} f(x)$$