Math 123, Practice Exam #2, November 7, 2003

1. Find the following:

   (a)
   $$\lim_{y \to \infty} \left(1 + \frac{2}{y}\right)^y$$

   (b)
   $$\lim_{x \to 0} \frac{\cos\left(\sqrt{5}\, x\right) - 1}{x^2}$$

   (c)
   $$\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right)$$

   (d) The horizontal and vertical asymptotes of
   $$y = \frac{4 - 3x}{\sqrt{16x^2 + 1}}$$

2. A man 6 feet tall is walking away from a light pole which is 30 feet high. If the tip of his shadow is moving at a rate equal to the distance between him and the light pole (in feet) then how fast is the man walking when he is 24 feet from the pole?

3. A spherical snowball is melting at a rate equal to its surface area. How fast is its radius shrinking when its volume is equal to its surface area?

4. Two nonnegative numbers are such that the sum of the first number and 3 times the second number equals 10. Find these numbers if the sum of their squares is as small as possible.

5. A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so that the total area enclosed is (a) a maximum? (b) A minimum?

6. Use Newton's method to find the solution to equation $\ln(4 - x^2) = x$ up to an accuracy of 5 decimal places.

7. Consider the function
   $$f(x) = 3x^4 - 4x^3 + 20000$$

   (a) On what interval(s) is f increasing?

   (b) On what interval(s) is f concave down?

   (c) Find the inflection point(s) of f.

(d) Find the critical points of f.

(e) Find the local maximum (maxima) of f.

(f) Find the global minimum of f on the interval $[-2, 3]$.

8. Repeat the previous problem for

$$f(x) = |x^2 - 5|$$

9. Suppose the graph on the following page is of $y = f(x)$

(a) Find the critical numbers of f.

(b) On what interval(s) is f increasing?

(c) On what interval(s) is f concave down?

(d) Find the values of x on the interval $(-\infty, \infty)$ where f has a local minimum.

(e) Find the values of x on the interval $[0, 4]$ where f has a global minimum.

(f) Find the x values of all inflection points of f.

10. Suppose the graph on the following page is of $y = f'(x)$ (**NOT** $f(x)$).

(a) Find the critical numbers of f.

(b) On what interval(s) is f increasing?

(c) On what interval(s) is f concave down?

(d) Find the values of x on the interval $(-\infty, \infty)$ where f has a local minimum.

(e) Find the values of x on the interval $[0, 4]$ where f has a global minimum.

(f) Find the x values of all inflection points of f.