Math 123, Practice Questions for Exam #3, December 3, 2003

1. Find the following:

   (a)
   $$\int (5t^3 - \frac{7}{t^4} + 6\sqrt{t} - 4\sin t)\, dt$$

   (b)
   $$\int (2e^x + \frac{3}{x} - \pi^x + 7\sec^2 x)\, dx$$

   (c)
   $$\int_1^2 \frac{1-x^3}{x^2}\, dx$$

   (d)
   $$\frac{d}{dx}\int_3^x \sqrt{t^3+1}\, dt$$

   (e)
   $$\int_0^{\sqrt{\pi}} \frac{d}{dt}\cos t^2\, dt$$

   (f)
   $$\int_{-1}^{10} f(x)\, dx$$

   where
   $$f(x) = \begin{cases} -x^2, & \text{if } x \le 0 \\ 2x & \text{if } 0 < x < 3 \\ -5 & \text{if } x \ge 3. \end{cases}$$

2. Find the following:

   (a)
   $$\frac{d}{dx}\int_{\sin x}^{x^3} e^{t^2}\, dt$$

   (b) $f(x)$ which satisfies the equation
   $$\int_1^x \frac{f(t)}{t}\, dt = 3x^{\frac{1}{3}} - 3$$

   (c) the average speed an object moving along a line between $-2 \le t \le 6$ if its velocity at time t is given by $v(t) = t^2 - 3t - 4$

3. Michelle begins walking along a line at time $t = 0$. Her acceleration at time $t \ge 0$ is $a(t) = 6t - 7$. Suppose that her initial velocity is 1 and her initial position is 3. If $s(t)$ denotes her position at time t and $v(t)$ denotes her velocity at time t then anser the following:

   (a) Find her velocity at $t = 4$.

   (b) Find her position at $t = 2$.

(c) When does she return to her starting position?

4. Consider the following Riemann sum:

$$I := \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)^8 \left(\frac{2}{n}\right).$$

(a) Write I as a definite integral.

(b) Calculate I (using any method you like).

5. Consider the graph below. Find the following:

(a)

$$\int_{-5}^{0} f(x)\,dx$$

(b) $F'(-3)$ where

$$F(x) := \int_{-6}^{x} f(t)\,dt$$

(c)

$$\int_{-3}^{1} |\,f(x)\,|\,dx$$

(d)

$$\int_{-3}^{4} f'(x)\,dx$$

(e)

$$\int_{-1}^{2} f'(x^2)\,x\,dx$$

(f)

$$\int_{5}^{7} (9(f(x))^2 - 8)\,dx$$

(g) Let $g(x) = \int_{-1}^{x} f(t)\,dt$. Consider only those x values $-4 \le x \le 1$. Find the values of x over which $g(x)$ is increasing. Find the values of x over which $g(x)$ is concave up.