Math 563A1, Final Exam, December 1, 2003
Prof. Takashi Kimura

This exam is due on Fri Dec 12 at noon in my office MCS 234. Please slide it under my door if I'm not there.

The exam is open book and you may work with others although you must write up the solutions yourself. Good luck!

1. (10 points) A surface S with the surface patch

$$\sigma(u, v) = (\cos u \cos v, \sin u \cos v, v + \sin u)$$

   is called the Pretzel Surface. (Plot is to see why it's called that.) Find its principal curvatures and principal directions. Find its Gaussian and Mean Curvatures.

2. (15 points) Exercises 9.2, 9.4, and 9.8 from the text.

3. (10 points) Find the lines of curvatures and the asymptotic curves of $z = xy$. See Exercise 6.18 for the definition of lines of curvature and Exercise 6.12 for the definition of asymptotic curves.

4. (10 points) Let $\alpha : I \to S$ be a curve parametrized by its arc length s which has everywhere nonzero curvature. Consider the parametrized surface

$$\sigma(s, v) := \alpha(s) + vb(s)$$

   where (s, v) belongs to $I \times (-\epsilon, \epsilon)$ for $\epsilon > 0$, I an open interval, and b is the binormal vector of α. Prove that if ϵ is small then image of σ, S, is a regular surface and that α is a geodesic of S.

5. (15 points) Exercises 11.1, 11.3, and 11.4 from the text.