MA 129 Discussion 10/25/2004
Takashi Kimura
The divergence of Taylor Series of ArcTangent about 0 when x =2

■ Here is the Taylor Polynomial of ArcTangent function up to order 2m+1 about 0

```
In[2]:= f[x_,m_] := Sum[ (-1)^n x^(2 n+1)/(2 n + 1),{n,0,m}]
```

```
In[3]:= f[x,5]
```

Out[3]= $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{x^{11}}{11}$

```
In[4]:= f[x,20]
```

Out[4]= $x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\frac{x^9}{9}-\frac{x^{11}}{11}+\frac{x^{13}}{13}-\frac{x^{15}}{15}+\frac{x^{17}}{17}-\frac{x^{19}}{19}+\frac{x^{21}}{21}-\frac{x^{23}}{23}+\frac{x^{25}}{25}-\frac{x^{27}}{27}+\frac{x^{29}}{29}-\frac{x^{31}}{31}+\frac{x^{33}}{33}-\frac{x^{35}}{35}+\frac{x^{37}}{37}-\frac{x^{39}}{39}+\frac{x^{41}}{41}$

■ Define a function which plugs in x=2 into the Taylor Polynomial of degree 2n+1 about 0 and then numerically evaluates it up to 100 decimal places

```
In[5]:= g[n_] := N[f[2,n],100]
```

■ Here's the result after plugging into the degree 21 Taylor Polynomial

```
In[6]:= g[10]
```

```
Out[6]= 78284.16853885708374872461559768061316048932148003355433695990971532767198401873324473\
        943668990108619
```

■ Here's the result after plugging into the degree 201 Taylor Polynomial -- it's getting big!

```
In[7]:= g[100]
```

Out[7]= $1.27659387449970066958699531537490737440887563794919320027388492019672851690533151520722523395297787 5\times 10^{58}$

■ Here's the result after plugging into the degree 2001 Taylor Polynomial -- it's getting really, really huge!

```
In[8]:= g[1000]

Out[8]= 9.178619060420475585091935144968522558012371260463826523090220591477221418034160040198\
        012628921199731 × 10^598
```

■ Meanwhile, the ArcTangent of 2 is...

```
In[9]:= ArcTan[2]

Out[9]= ArcTan[2]

In[10]:= N[%]

Out[10]= 1.10715
```

■ ... which is not even close! So the Taylor Series of ArcTangent about 0 of degree 2n+1 becomes arbitrarily large (diverges) when one plugs in 2 as n becomes arbitrarily large.

■ Now let's calculate the first few digits of Pi using the ArcTangent Taylor Series

```
In[11]:= g[n_] := N[4 f[1,n],100]

In[12]:= g[3]

Out[12]= 2.895238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095238095
```

```
In[15]:= g[10000000]

Out[15]= 3.1415927535897832384633933832545028823221693368752042584747883369054726558960773872 9\
          9908782794192263
```

■ As you can see, it converges to the correct answer slowly which is, out to 100 digits:

```
In[16]:= N[Pi,100]

Out[16]= 3.14159265358979323846264338327950288419716939937510582097494459230781640628620899862\
          8034825342117068
```