Math 123, Practice Exam Questions for Exam #1, September 29, 1999

Unless explicitly stated, full credit will be awarded only if the solution is correct and the steps leading up to it are correct.

1. Calculate the following:

   (a)
   $$\lim_{x \to -1} \frac{x^2 - 4x - 5}{x + 1}$$

   (b)
   $$\lim_{x \to 3} \frac{5x^2}{2x - 1}$$

   (c)
   $$\lim_{x \to 2^+} \frac{4 - x^2}{|2 - x|}$$

   (d)
   $$\lim_{x \to 1} \sqrt{\frac{2x^3 - 3x + 5}{2 - x}}$$

   (e)
   $$\lim_{x \to 3} f(x) \quad \text{where} \quad f(x) = \begin{cases} x^2 & \text{if } x > 3 \\ 8 & \text{if } x = 3 \\ 12 - x & \text{if } x < 3 \end{cases}$$

   (f)
   $$\lim_{x \to 3^-} \frac{x + 3}{x^2 - 9}$$

2. Consider the function
   $$f(x) = \begin{cases} x - c, & \text{if } x > 2; \\ 3x^2, & \text{if } x \leq 2 \end{cases}$$
   where c is a real number.

   (a) What value of c makes the function f continuous everywhere?

   (b) Calculate $f'(7)$

   (c) Calculate $f'(-1)$

   (d) Calculate $f'(2)$

3. Compute the derivative of the following functions:

   (a)
   $$f(x) = \pi^4$$

   (b)
   $$f(x) = 3x^5 - x^2 + 9$$

(c)

$$f(x) = \frac{2}{x^2} - 3\sqrt{x}$$

(d)

$$f(x) = \frac{x^2}{2x - 3}$$

(e)

$$f(x) = x^2 e^x$$

(f)

$$f(t) = t \sin t$$

4. Find the equation for the tangent line to the curve $y = f(x)$ where through the point $(1, -3)$ where

$$f(x) = x^8 - 4x.$$

5. Suppose an object is moving along the real line with its position at time t given by the function $s(t) = \frac{1}{3}t^3 - 3t^2 - 7t + 10$.

   (a) When is the object at rest?

   (b) When is the object decelerating?

   (c) Find the velocity of the object at time $t = 2$.

6. Consider the function

$$f(x) = x^2 - \sin x$$

   Using your calculator, find the approximate value(s) of x (out to two decimal places) at which the tangent line to the graph $y = f(x)$ is horizontal.

7. Consider the graph of $y = f(x)$ on the next page.

   (a) Where is f undefined?

   (b) Where is f not continuous?

   (c) Where is f not differentiable?

   (d) On what interval(s) is f' positive? Where does f' vanish?

   (e) On what interval(s) is f concave down?

   (f) What are

   i.

$$f'(6)$$

   ii.

$$\lim_{x \to -5^+} f(x)$$

   iii.

$$\lim_{x \to -1} f(x)$$

   iv.

$$\lim_{x \to -\infty} f(x)$$