Math 123, Practice Exam #2, October 29, 1999

1. Find the following:

   (a)
   $$\lim_{y \to \infty} \left(1 + \frac{2}{y}\right)^{y}$$

   (b)
   $$\lim_{x \to 0} \frac{\cos\left(\sqrt{5}\,x\right) - 1}{x^2}$$

   (c) The horizontal and vertical asymptotes of
   $$y = \frac{4 - 3x}{\sqrt{16x^2 + 1}}$$

   (d) $f'(x)$ where
   $$f(x) = \sin\left(x^{100}\right)$$

   (e) $f'(t)$ where
   $$f(t) = \tan\left(\sqrt{t^4 + 2}\right)$$

   (f) $f'(x)$ where
   $$f(x) = \frac{\ln x}{x}$$

   (g) $f'(x)$ where
   $$f(x) = x^{x^2}$$

   (h) $f'(x)$ where
   $$f(x) = 10^{\cos x}$$

   (i) $f'(x)$ where
   $$f(x) = \arctan(x^3)$$

2. Find the equation for the tangent line to the curve given by the equation $\cos(xy) - 3y^3 = e^x + 1$ through the point $(0, -1)$.

3. We wish to find an approximate value of the positive root of $2\sin x - x = 0$ using Newton's Method.

   (a) Find the formula for x_{n+1} in terms of x_n.

   (b) Find the positive root (up to 5 decimal places) of this equation using Newton's method using the initial value $x_1 = 1$. Make a table of all x_n values which you need to produce your answer.

4. A man 6 feet tall is walking away from a light pole which is 30 feet high. If the tip of his shadow is moving at a rate equal to the distance between him and the light pole (in feet) then how fast is the man walking when he is 24 feet from the pole?

5. A spherical snowball is melting at a rate equal to its surface area. How fast is its radius shrinking when its volume is equal to its surface area?

6. Solve the following:

   (a) Find the area of the largest rectangle that can be inscribed in a semicircle of radius r.

(b) Two nonnegative numbers are such that the sum of the first number and 3 times the second number equals 10. Find these numbers if the sum of their squares is as small as possible.

7. Consider the functon

$$f(x) = 1 + x - 3x^{\frac{2}{3}}$$

   (a) Find the critical point(s) of f.
   (b) On what interval(s) is f increasing?
   (c) On what interval(s) is f concave down?
   (d) Find the inflection point(s) of f.
   (e) Find the local minimum (minima) of f.
   (f) Find the global maximum of f on the interval $[-2, 3]$

8. Suppose the graph on the next page is $y = f(x)$.
   (a) Find the interval(s) where f is increasing.
   (b) Find the interval(s) where f is concave up.
   (c) Find the inflection point(s) of f.
   (d) Find the critical numbers of f.
   (e) Find the local maximum (maxima) of f.
   (f) Find the horizontal asymptotes of f.

9. Suppose the graph on the next page is $y = f'(x)$ where we assume that f is continuous everywhere.
   (a) Find the interval(s) where f is increasing.
   (b) Find the interval(s) where f is concave up.
   (c) Find the inflection point(s) of f.
   (d) Find the critical numbers of f.
   (e) Find the local maximum (maxima) of f.
   (f) Find $f''(7)$.