Math 123, Practice Exam #3, December 6, 1999

1. Find the following:

(a)

$$\int (5t^3 - \frac{7}{t^4} + 6\sqrt{t} - 4\sin t)\, dt$$

(b)

$$\int \sin^{100}(x)\cos(x)\, dx$$

(c)

$$\int \frac{3x^2 - 16x + 5}{\sqrt{x^3 - 8x^2 + 5x + 3}}\, dx$$

(d)

$$\int (x - \frac{3}{2})\sin(x^2 - 3x)\, dx$$

(e)

$$\int x \ln x^3\, dx$$

(f)

$$\int_1^2 \frac{1 - x^3}{x^2}\, dx$$

(g)

$$\int_0^{\frac{\pi}{4}} \tan^5 x \sec^2 x\, dx$$

(h)

$$\frac{d}{dx}\int_3^x \sqrt{t^3 + 1}\, dt$$

(i)

$$\frac{d}{dx}\int_{2x}^{x^3} \sin t^2\, dt$$

(j)

$$\int_0^{\sqrt{\pi}} \frac{d}{dt}\cos t^2\, dt$$

(k)

$$\int_2^4 f'(x)\sin(f(x))dx$$

if f is a continuous function on the interval $[0, 20]$ such that $f(0) = 3$, $f(2) = 1$, $f(4) = 7$, and $f(20) = 5$.

(l)

$$\int_1^3 (2x-8)e^{-x}\,dx$$

(m)

$$\int_{-1}^{10} f(x)\,dx$$

where

$$f(x) = \begin{cases} -x^2, & \text{if } x \le 0 \\ 2x & \text{if } 0 < x < 3 \\ -5 & \text{if } x \ge 3. \end{cases}$$

2. John moves along the real line with velocity at time t given by $v(t) = \sin^3 t$ and his position at time t is denoted by $s(t)$. Furthermore at $t = 0$, John is standing at $s(0) = 5$.
   (a) Find John's position at $t = \frac{5}{4}\pi$.
   (b) Find John's displacement between times $t = 0$ and $t = \frac{5}{4}\pi$.
   (c) Find the total distance traveled by John between times $t = 0$ and $t = \frac{5}{4}\pi$.

3. Consider the following Riemann sum:

$$I := \lim_{n\to\infty} \sum_{i=1}^{n} \left(1 + \frac{2i}{n}\right)^8 \left(\frac{2}{n}\right).$$

   (a) Write I as a definite integral.
   (b) Calculate I (using any method you like).

4. Calculate the area of the region bounded by the x-axis, and $y = -x^2 + 4x + 5$ which is to the left of the line $x = \pi$.

5. Consider the graph below. Find the following:
   (a)

$$\int_{-5}^{0} f(x)\,dx$$

   (b) $F'(-1)$ where

$$F(x) := \int_{-3}^{x^4} f(t)\,dt$$

   (c)

$$\int_{-3}^{1} |\,f(x)\,|\,dx$$

   (d)

$$\int_{-3}^{4} f'(x)\,dx$$

   (e)

$$\int_{-1}^{2} f'(x^2)\,x\,dx$$

   (f)

$$\int_{5}^{7} (9(f(x))^2 - 8)\,dx$$