Math 124, Practice Questions for Exam #2, April 19, 2000

1. Which of the following series converges? Explain your answer.

   (a)
   $$\frac{1}{\ln 2} - \frac{1}{\ln 3} + \frac{1}{\ln 4} - \frac{1}{\ln 5} + \cdots$$

   (b)
   $$\sum_{n=1}^{\infty} \frac{n}{(\sin n)^2}$$

   (c)
   $$\sum_{n=1}^{\infty} \frac{n^2}{8n^7 + 6n^2 + 5}$$

   (d)
   $$\sum_{n=1}^{\infty} 2^{2n+1} 5^{-n}$$

2. Consider the following series
   $$s = \sum_{n=0}^{\infty} \frac{(-1)^n}{1+n^2}$$
   How many terms in the series must one sum up in order to obtain s correct to within 0.000001 accuracy?

3. Consider the following series
   $$s = \sum_{n=1}^{\infty} \frac{1}{n^3}$$
   How many terms in the series must one sum up in order to obtain s correct to within an accuracy of 0.00001?

4. Consider the function $f(x) = \frac{3x^4}{5x-7}$.
   (a) Write $f(x)$ as a power series.
   (b) Find its radius of convergence.
   (c) Find its interval of convergence.

5. Consider the function $f(x) = \tan^{-1}(x^3)$.
   (a) Write $f(x)$ as a power series.
   (b) Find its radius of convergence.

6. Consider the series $\sum_{k=1}^{\infty} \frac{x^k}{2^k k}$.
   (a) Find its radius of convergence.
   (b) Find its interval of convergence.

7. Find the Taylor series centered at 1 of the function $f(x) = x^{2/3}$.

8. (a) Find the MacLauren series of the function $f(x) = \ln(3+x)$.
   (b) Find its radius of convergence.