

Math 124, Practice Questions for the Final Exam, May 5, 2000

1. Calculate the following:

(a)

$$\int \frac{\cos x}{\sqrt{\sin x}} dx$$

(b)

$$\int e^{x^6-x^3} \left(x^5 - \frac{x^2}{2}\right) dx$$

(c)

$$\int x^2 \sqrt{3x^3 + 1} dx$$

(d)

$$\int \frac{1}{x \ln x} dx$$

(e)

$$\int x \sin x dx$$

(f)

$$\int x e^{-2x} dx$$

(g) The average value of \bar{f} of the function $f(x) = x^{200}$ over the interval $[1, 12]$

(h)

$$\int_1^{\infty} \frac{1}{x^2}$$

2. Calculate the area of the region in the xy -plane bounded by the curves $y = 10 - 3x$ and $y = x^2$.

3. Let R be the region in the xy -plane bounded by the y -axis, the x -axis, and $y = 1 - 5x$. Find the centroid of R .

4. Find the arclength of the curve $y = 2 + (x + 2)^{\frac{3}{2}}$ between the points $(-2, 2)$ and $(2, 10)$.

5. Let R be the region in the plane bounded by the curves $y = x$ and $y = 4 - (x - 2)^2$. Calculate the volume of the region obtained by revolving R around the y -axis.

6. (a) Does the following series converge? Explain your answer.

$$\sum_{n=1}^{\infty} (-1)^n \frac{(\sin n)^2}{n^2}$$

(b) Consider the series

$$\sum_{n=0}^{\infty} 8e^{-2n}$$

i. Does it converge?

ii. What number does it converge to?

(c) Does the following series converge? Explain your answer.

$$\sum_{n=1}^{\infty} \frac{7n^3 + 6n + 1}{9n^8 + n^2 + 10}$$

(d) Does the following series converge? Explain your answer.

$$\sum_{n=1}^{\infty} \frac{n^3}{n!}$$

7. Consider the series

$$\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^n n^2}.$$

(a) Find its radius of convergence.

(b) Find its interval of convergence.

8. Find the Taylor series centered at 1 of the function $f(x) = \sin x$ about $x = \frac{\pi}{4}$.