**Math 124, Practice Exam Questions for Exam #1, February 22, 2002**

1. Find the following:

(a)

$$\int \sin^{100}(x)\cos(x)\,dx$$

(b)

$$\int \frac{3x^2-16x+5}{\sqrt{x^3-8x^2+5x+3}}\,dx$$

(c)

$$\int (x-\frac{3}{2})\sin(x^2-3x)\,dx$$

(d)

$$\int x\ln x^3\,dx$$

(e)

$$\int e^{\sqrt{x}}dx$$

(f)

$$\int_0^{\frac{\pi}{4}} \tan^5 x\sec^2 x\,dx$$

(g)

$$\int_2^4 f'(x)\sin(f(x))dx$$

if $f$ is a continuous function on the interval $[0,20]$ such that $f(0)=3$, $f(2)=1$, $f(4)=7$, and $f(20)=5$.

(h)

$$\int_1^3 (2x-8)e^{-x}\,dx$$

(i)

$$\int_{-\infty}^3 \frac{1}{1+x^2}\,dx$$

(j)

$$\int_1^5 \frac{x}{x^2-4}\,dx$$

(k)

$$\int_0^1 x^s\ln x\,dx$$

where $s$ is a constant not equal to $-1$

(l) $\bar{f}$, the average value of $f(x) = x^2$ over the interval $[3, 8]$.

2. Consider the region $R$ in the xy-plane where $x \geq 0$ bounded by the graphs $y = x^3$ and $y = x^5$.
   (a) Calculate the area of $R$.
   (b) Consider the solid $S$ whose base is $R$ and whose cross-sections perpendicular to the $x$-axis are equilateral triangles. Find the volume $V$ of $S$.
3. Consider the region $R$ in the xy-plane bounded by $y = 0$, and $y = 9 - x^2$. Find the centroid of $R$.
4. Find the arc length of the part of the curve $y = 2x^{\frac{3}{2}}$ between the points $(1, 2)$ and $(4, 16)$.
5. Consider the region $Q$ in the xy-plane bounded by the graphs $y = x$ and $y = (x-2)^2$. Find the volume of the solid obtained by revolving $Q$ about the $x$-axis.
6. Consider the differential equation
$$\frac{dy}{dx} + 2y = e^x.$$
   (a) Verify that
$$y = Ce^{-2x} + \frac{1}{3}e^x$$
   is a solution for any constant $C$.
   (b) Find the solution which satisfies $y(0) = 8$.
7. Consider the differential equation
$$y^2 \frac{dy}{dx} + 3\cos x = 0$$
   (a) Find the general solution to the above equation.
   (b) Find the particular solution which satisfies $y(\pi) = 2$.