

**Math 124, Practice Questions for Exam #2, April 24, 2002**

1. Does the following series converge? Explain your answer.

(a)

$$\frac{1}{\ln 2} - \frac{1}{\ln 3} + \frac{1}{\ln 4} - \frac{1}{\ln 5} + \cdots$$

(b)

$$\sum_{n=1}^{\infty} \frac{n}{(\sin n)^2}$$

(c)

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

(d)

$$\sum_{n=1}^{\infty} \frac{n^2}{8n^7 + 6n^2 + 5}$$

(e)

$$\sum_{n=1}^{\infty} 2^{2n+1} 5^{-n}$$

(f)

$$\sum_{n=1}^{\infty} a_n$$

where  $a_1 = 7$  and  $a_{n+1} = \frac{2n+3}{5n+9}a_n$  for all  $n \geq 1$ .

2. Consider the following series

$$s = \sum_{n=0}^{\infty} \frac{(-1)^n}{1+n^2}$$

How many terms in the series must one sum up in order to obtain  $s$  correct to within 0.000001 accuracy?

3. Consider the following series

$$s = \sum_{n=1}^{\infty} \frac{1}{n^3}$$

How many terms in the series must one sum up in order to obtain  $s$  correct to within an accuracy of 0.00001?

4. Consider the function  $f(x) = \frac{3x^4}{5x-7}$ .

- (a) Write  $f(x)$  as a power series.
- (b) Find its radius of convergence.
- (c) Find its interval of convergence.

5. Consider the function  $f(x) = \tan^{-1}(x^3)$ .

- (a) Write  $f(x)$  as a power series.
  - (b) Find its radius of convergence.
6. Consider the series  $\sum_{k=1}^{\infty} \frac{x^k}{2^k k}$ .
- (a) Find its radius of convergence.
  - (b) Find its interval of convergence.
7. Find the Taylor series centered at 1 of the function  $f(x) = x^{2/3}$ .
8. (a) Find the MacLauren series of the function  $f(x) = \ln(3+x)$ .
- (b) Find its radius of convergence.
9. (a) Find a power series expression for the following integral:

$$\int e^{-x^4} dx$$

- (b) Find a series representation for the following:

$$\int_0^2 e^{-x^4} dx$$

10. Calculate

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)! 2^{2n}}$$