Math 124, Practice Exam Questions for Exam #1, February 20, 2004

1. Find the following:

(a)

$$\int \sin^{100}(x)\cos(x)\,dx$$

(b)

$$\int \frac{3x^2 - 16x + 5}{\sqrt{x^3 - 8x^2 + 5x + 3}}\,dx$$

(c)

$$\int (x - \frac{3}{2})\sin(x^2 - 3x)\,dx$$

(d)

$$\int x \ln x^3\,dx$$

(e)

$$\int e^{\sqrt{x}} dx$$

(f)

$$\int_0^{\frac{\pi}{4}} \tan^5 x \sec^2 x\,dx$$

(g)

$$\int_2^4 f'(x)\sin(f(x))dx$$

if f is a continuous function on the interval $[0, 20]$ such that $f(0) = 3$, $f(2) = 1$, $f(4) = 7$, and $f(20) = 5$.

(h)

$$\int_1^3 (2x - 8)e^{-x}\,dx$$

(i)

$$\int_{-\infty}^3 \frac{1}{1 + x^2}\,dx$$

(j)

$$\int_1^5 \frac{x}{x^2 - 4}\,dx$$

(k)

$$\int_0^1 x^s \ln x\,dx$$

where s is a constant not equal to -1

(l) $\bar{f}$, the average value of $f(x) = x^2$ over the interval $[3, 8]$.

2. Consider the region R in the xy-plane where $x \geq 0$ bounded by the graphs $y = x^3$ and $y = x^5$.

   (a) Calculate the area of R.

   (b) Consider the solid S whose base is R and whose cross-sections perpendicular to the x-axis are equilateral triangles. Find the volume V of S.

3. Consider the region R in the xy-plane bounded by $y = 0$, and $y = 9 - x^2$. Find the centroid of R.

4. Find the arc length of the part of the curve $y = 2x^{\frac{3}{2}}$ between the points $(1, 2)$ and $(4, 16)$.

5. Find the arc length of the part of the parametric curve $x = e^t \cos t$ and $y = e^t \sin t$ between the points $(1, 0)$ and $(-e^\pi, 0)$.

6. Consider the region Q in the xy-plane bounded by the graphs $y = x$ and $y = (x - 2)^2$. Find the volume of the solid obtained by revolving Q about the x-axis.

7. Consider the differential equation

$$\frac{dy}{dx} + 2y = e^x.$$

   (a) Verify that

$$y = Ce^{-2x} + \frac{1}{3}e^x$$

   is a solution for any constant C.

   (b) Find the solution which satisfies $y(0) = 8$.

8. Consider the differential equation

$$y^2 \frac{dy}{dx} + 3\cos x = 0$$

   (a) Find the general solution to the above equation.

   (b) Find the particular solution which satisfies $y(\pi) = 2$.