Math 564, Midterm Exam, March 17, 2004
Due in class on March 24, 2004
Prof. Takashi Kimura

Throughout, $\mathbf{R}$ is the set of real numbers, $\mathbf{Q}$ is the set of rational numbers and $\mathbf{Z}$ is the set of integers.

1. (10 points) Let X be a set. Let T and T' be two topologies on X such that $T \subseteq T'$. What does the compactness of X under one of these topologies imply about compactness under the other?

2. (10 points) Let $f : X \to Y$ be a continuous function where X is compact and Y is Hausdorff. Prove that f is a closed map, *i.e.* f maps closed sets to closed sets.

3. (10 points) Consider the set $\mathbf{R}$ of real numbers. Consider the subset
$$K := \{ \frac{1}{n} \mid n > 0 \text{ and } n \in \mathbf{Z} \}.$$
   (a) Let $\mathbf{R}$ be given the discrete topology. Find the closure of K in $\mathbf{R}$ in this topology.
   (b) Let $\mathbf{R}$ be given the topology arising from the basis $\mathcal{B} := \{ [a, b) \mid a < b \}$. Find the closure of K in $\mathbf{R}$ in this topology.
   (c) Let $\mathbf{R}$ be given the topology arising from the basis $\mathcal{B}' := \{ (a, b] \mid a < b \}$. Find the closure of K in $\mathbf{R}$ in this topology.

4. (10 points) Prove that $\mathbf{R}$ is not homeomorphic to $\mathbf{R}^k$ for all $k \geq 2$.

5. (10 points) Problem #10 from Chapter 3 in our text.

6. (10 points) Problem #26 from Chapter 3 in our text.

7. (10 points) Define an equivalence relation on the plane $X := \mathbf{R}^2$ as follows: $(x_0, y_0) \sim (x_1, y_1)$ if and only if
$$x_0 + y_0^2 = x_1 + y_1^2.$$
Let X^* denote the collection of equivalence classes, in the quotient topology. X^* is homeomorphic to a familiar space. What is it?

8. (10 points) Let f and g both be continuous functions $X \to CY$ where CY is the cone of Y. Show that f and g are homotopic.

9. (10 points) Problem #9 from Chapter 5 in our text.

10. (10 points) Problem #13 from Chapter 5 in our text.