MATHEMATICS 727 A1 Introduction to Algebraic Topology Fall Semester 2005 Instructor: Takashi Kimura e-mail: kimura@math.bu.edu Phone: (617)353-1486 Office: MCS 234

Lectures: MWF 1-2 in MCS B23

Office Hours: MW 2-3, F 10-11

Text: Elements of Algebraic Topology, by J. Munkres, ISBN 0201627280.

Web: http://math.bu.edu/people/kimura/Teaching/Fall05/727/

Content: One of the central questions in topology is the following: Given two topological spaces, is there an effective way to determine whether or not they are homeomorphic? This turns out to be a very difficult question. Algebraic topology approaches this question in a simple way. Given any topological space, assign an algebraic structure (say, a ring) to it a systematic way so that the rings are the isomorphic if the two spaces are homeomorphic. One way to show that two space are not homeomorphic, then, would be to prove that their associated rings are not isomorphic.

The course in basic topology has already introduced one such assignment, namely, the fundamental group which assigns a group to a topological space. Generalizations of the fundamental group give rise to the subject of homotopy theory which assigns homotopy groups to topological spaces. While these objects are fairly easy to define, they are often quite difficult to compute.

Another such assignment gives rise to homology/cohomology theory which assigns an abelian group to a topological space (called its homology/cohomology group). While this object is trickier to define, it is often easier to compute. Homology/cohomology theory allows one to define numerical invariants such as the Euler characteristic for a large class of topological spaces. This course will primarily serve as an introduction to homology/cohomology theory.

- **Prerequisites:** This course assumes an understanding of basic notions from topology, e.g. topological spaces, compactness, connectedness, metric spaces and continuity. Some knowledge of algebra, e.g. abelian groups and rings, will be helpful although it will be introduced as necessary.
- **Homework:** Homework will be assigned periodically. Students may discuss homework with each other (and are encouraged to do so) but all written work must be prepared independently.