

Math 563A1, Homework 1
Due September 18, 2006
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1. (10 points) Let a, b, c belong to $\mathbf{R}^3$. Prove the following identities:

(a)

$$a \cdot (b \times c) = -b \cdot (a \times c)$$

(b)

$$a \times (b \times c) = (a \cdot c)b - (a \cdot b)c$$

2. (10 points) Consider the curve

$$\beta(s) = \left(\frac{1}{3}(1+s)^{\frac{3}{2}}, \frac{1}{3}(1-s)^{\frac{3}{2}}, \frac{s}{\sqrt{2}} \right)$$

where $-1 < s < 1$.

(a) Show that β has unit speed.

(b) Show that

$$\kappa = \frac{1}{\sqrt{8(1-s^2)}}.$$

(c) Show that

$$N = \left(\sqrt{\frac{1-s}{2}}, \sqrt{\frac{1+s}{2}}, 0 \right)$$

and

$$B = T \times N = \left(-\frac{1}{2}\sqrt{1+s}, \frac{1}{2}\sqrt{1-s}, \frac{1}{\sqrt{2}} \right).$$

(d) Show that $\tau = \kappa$.

3. (10 points) Compute T , N , B , κ , and τ for the curve $\beta(t) = (e^t \cos t, e^t \sin t, e^t)$.

4. (10 points) If a rigid body moves along a (unit speed) curve $\alpha(s)$ then the motion of the body consists of a translation along α and rotation about α . The rotation is determined by an angular velocity vector ω , called the Darboux vector, which satisfies the equations $T' = \omega \times T$, $N' = \omega \times N$, and $B' = \omega \times B$.

(a) Show that ω , in terms of T , N , and B is given by $\omega = \tau T + \kappa B$. Hint: Write $\omega = aT + bN + cB$ and take cross products with T , N , and B to determine a, b, c .

(b) Show that $T' \times T'' = \kappa^2 \omega$.