Math 563A1, Homework 2
Due October 9, 2007
Prof. Takashi Kimura

1. (10 points) Pressley Problem 4.16.

2. (10 points) Show that the hyperboloid of one sheet S is a smooth surface where S consists of points (x, y, z) such that

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1.$$

3. (10 points) Pressley Problem 4.19.

4. (20 points) Consider a circle of radius r in the xz-plane whose center lies on the x-axis a distance R from the origin where $r < R$. Rotate this circle about the z-axis to obtain a torus S.

   (a) Show that S has an atlas consisting of surface patches of the form

$$\sigma(u, v) := ((R + r\cos u)\cos v, (R + r\cos u)\sin v, r\sin u)$$

   where (u, v) belong to suitable open subsets of $\mathbf{R}^2$.

   (b) Calculate the standard unit normal N_σ.

   (c) Find the equation for T_pS, the tangent plane to S at the point p where $p := (R/\sqrt{2}, R/\sqrt{2}, r)$.

   (d) Calculate the first fundamental form of σ.