Math 563A1, Homework 5
Due December 10, 2007
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1. (10 points) Let γ be a path contained on a surface patch σ. Let $\gamma^\tau(t)$ denote a smooth family of curves on σ through given points p and q, i.e., it is defined as in paragraph 2 of page 191 of Pressley. Consider the function

$$S(\tau) := \frac{1}{2} \int_a^b |\dot{\gamma}^\tau(t)|^2 dt$$

   where dot denotes $\frac{d}{dt}$. Prove that γ is a geodesic if and only if

$$\frac{d}{d\tau} S(\tau) = 0$$

   when $\tau = 0$ for all families γ^τ with $\gamma^0 = \gamma$.

2. (10 points) Let $\sigma(u, v)$ be a conformal surface patch where (u, v) belong to an open set U in $\mathbf{R}^2$. Prove that

$$\sigma_{uu} + \sigma_{vv} = 2EH\nu$$

   where ν is the standard unit normal vector of σ, H is the mean curvature, and E is the usual component of the first fundamental form.

3. (10 points) Pressley Problem 9.2

4. (10 points) Pressley Problem 9.4

5. (10 points) Pressley Problem 10.1