Math 124, Practice Questions for Exam #2, April 21, 2006

1. Does the following series converge? Explain your answer.

   (a)

   $$\frac{1}{\ln 2} - \frac{1}{\ln 3} + \frac{1}{\ln 4} - \frac{1}{\ln 5} + \cdots$$

   (b)

   $$\sum_{n=1}^{\infty} \frac{n}{(\sin n)^2}$$

   (c)

   $$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

   (d)

   $$\sum_{n=1}^{\infty} \frac{n^2}{8n^7 + 6n^2 + 5}$$

   (e)

   $$\sum_{n=1}^{\infty} 2^{2n+1} 5^{-n}$$

   (f)

   $$\sum_{n=1}^{\infty} a_n$$

   where $a_1 = 7$ and $a_{n+1} = \frac{2n+3}{5n+9} a_n$ for all $n \geq 1$.

2. Consider the following series

   $$s = \sum_{n=0}^{\infty} \frac{(-1)^n}{1+n^2}$$

   How many terms in the series must one sum up in order to obtain s correct to within 0.000001 accuracy?

3. Consider the following series

   $$s = \sum_{n=1}^{\infty} \frac{1}{n^3}$$

   How many terms in the series must one sum up in order to obtain s correct to within an accuracy of 0.00001?

4. Suppose that we know that a power series $\sum_{n=0}^{\infty} c_n (x-1)^n$ converges when $x = 4$ and diverges at $x = -6$.

   (a) What can one say about the convergence of the series at $x = -1$?

   (b) What can one say about the convergence of the series at $x = -7$?

   (c) What can one say about the convergence of the series at $x = -2$?

5. Consider the function $f(x) = \frac{3x^4}{5x-7}$.

(a) Write $f(x)$ as a power series.
(b) Find its radius of convergence.
(c) Find its interval of convergence.

6. Consider the function $f(x) = \tan^{-1}(x^3)$.

   (a) Write $f(x)$ as a power series.
   (b) Find its radius of convergence.

7. Consider the series $\sum_{k=1}^{\infty} \frac{x^k}{2^k k}$.

   (a) Find its radius of convergence.
   (b) Find its interval of convergence.

8. Find the Taylor series centered at 1 of the function $f(x) = x^{2/3}$.

9. (a) Find the MacLauren series of the function $f(x) = \ln(3 + x)$.
   (b) Find its radius of convergence.

10. (a) Find a power series expression for the following integral:

$$\int e^{-x^4}\, dx$$

   (b) Find a series representation for the following:

$$\int_0^2 e^{-x^4}\, dx$$

11. Calculate

$$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!\, 2^{2n}}$$