

**Math 124, Practice Exam Questions for Exam #1, February 23, 2007**

1. Find the following:

(a)

$$\int \sin^{100}(x) \cos^3(x) \, dx$$

(b)

$$\int \frac{3x^2 - 16x + 5}{\sqrt{x^3 - 8x^2 + 5x + 3}} \, dx$$

(c)

$$\int \left(x - \frac{3}{2}\right) \sin(x^2 - 3x) \, dx$$

(d)

$$\int x \ln x^3 \, dx$$

(e)

$$\int e^{\sqrt{x}} \, dx$$

(f)

$$\int_0^{\frac{\pi}{4}} \tan^5 x \sec^2 x \, dx$$

(g)

$$\int_2^4 f'(x) \sin(f(x)) \, dx$$

if  $f$  is a differentiable function on the interval  $[0, 20]$  such that  $f(0) = 3$ ,  $f(2) = 1$ ,  $f(4) = 7$ , and  $f(20) = 5$ .

(h)

$$\int_1^3 (2x - 8)e^{-x} \, dx$$

(i)

$$\int_{-\infty}^3 \frac{1}{1+x^2} \, dx$$

(j)

$$\int_1^5 \frac{x}{x^2 - 4} \, dx$$

(k)

$$\int_0^1 x^s \ln x \, dx$$

where  $s$  is a constant not equal to  $-1$

- (l)  $\bar{f}$ , the average value of  $f(x) = x^2$  over the interval  $[3, 8]$ .
2. Consider the region  $R$  in the  $xy$ -plane where  $x \geq 0$  bounded by the graphs  $y = x^3$  and  $y = x^5$ .
  - (a) Calculate the area of  $R$ .
  - (b) Consider the solid  $S$  whose base is  $R$  and whose cross-sections perpendicular to the  $x$ -axis are equilateral triangles. Find the volume  $V$  of  $S$ .
3. Consider the region  $R$  in the  $xy$ -plane bounded by  $y = 0$ , and  $y = 9 - x^2$ . Find the centroid of  $R$ .
4. Find the arc length of the part of the curve  $y = 2x^{\frac{3}{2}}$  between the points  $(1, 2)$  and  $(4, 16)$ .
5. Consider the region  $Q$  in the  $xy$ -plane bounded by the graphs  $y = x$  and  $y = (x - 2)^2$ . Find the volume of the solid obtained by revolving  $Q$  about the  $x$ -axis.
6. Consider the differential equation

$$\frac{dy}{dx} + 2y = e^x.$$

- (a) Verify that

$$y = Ce^{-2x} + \frac{1}{3}e^x$$

is a solution for any constant  $C$ .

- (b) Find the solution which satisfies  $y(0) = 8$ .
7. Consider the differential equation
 
$$y^2 \frac{dy}{dx} + 3 \cos x = 0$$
  - (a) Find the general solution to the above equation.
  - (b) Find the particular solution which satisfies  $y(\pi) = 2$ .
8. Find the orthogonal trajectories of the following family of curves:

$$y = \frac{k}{x}.$$