MATHEMATICS 726 A1
Differential Geometry II
Spring Semester 2007

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Lectures: MWF 11-12 in MCS B29
My Office Hours: TBA
Web Page: http://math.bu.edu/people/kimura/Teaching/Spring07/726/
Some References:

(1) *An Introduction to Lie Groups and Symplectic Geometry* by R. Bryant available at
http://www.math.duke.edu/~bryant/ParkCityLectures.pdf

(2) *Lectures on Symplectic Geometry* by Ana Cannas da Silva, *Lecture Notes in Mathematics* **1764**, Springer.

(3) *Moment maps, Cobordisms, and Hamiltonian Group Actions* by V. L. Ginzburg, V. Guillemin, and Y. Karshon, American Mathematical Society, ISBN: 0821805029. A PDF version is available at
http://www.ma.huji.ac.il/~karshon/monograph/index-pdf.html

(4) *Moment maps and combinatorial invariants of Hamiltonian T^n-spaces* by V. Guillemin, Birkhaüser, ISBN: 0817637702.

(5) *Operads in Algebra, Topology and Physics* by M. Markl, S. Schneider, J. Stasheff, American Mathematical Society, ISBN: 0821821342.

Content: This class is a continuation of MA 725 which focuses upon ideas related to symplectic geometry and its applications. A symplectic manifold (M, ω) is a smooth manifold M with a symplectic form, a closed, nondegenerate 2-form ω. Mathematically, symplectic geometry is an elegant subject which has a more topological feel than Riemannian geometry because of the lack of local invariants (such as a symplectic analog of curvature). It is also the subject of active current research in geometry, e.g. Gromov-Witten invariants are invariants of symplectic manifolds.

From the viewpoint of physics, symplectic manifolds are the framework for the coordinate invariant description of Hamiltonian classical mechanics. More succinctly, a symplectic manifold may be viewed as a "phase space." Hamiltonian systems with symmetries have associated "conserved quantities" and quotienting out by these symmetries yields a "reduced phase space." The latter has a coordinate invariant description in terms of symplectic reduction. Quantum mechanics can also be formulated in this language through geometric quantization, a coordinate invariant description of "canonical quantization."

We will study symplectic manifolds, coisotropic and Lagrangian submanifolds, symplecic manifolds with symmetry, Hamiltonian group actions, equivariant moment maps, and symplectic reduction. We will also cover symplectic manifolds with a torus action, the associated Delzant polytope, its combinatorial invariants, and the relationship with toric varieties. If time permits we will cover symplectic orbifolds and orbifold cohomology.

Prerequisites: This course assumes the material in Differential Geometry I (MA 725), Differential Topology I and II (MA 721 and MA 722). The material in Algebraic Topology (MA 727 and MA 728) and Lie Groups (MA 731) will also be helpful but is not necessary.

Homework: Homework will be assigned periodically. Students may discuss homework with each other (and are encouraged to do so) but all written work must be prepared independently.