Math 124, Practice Exam Questions for Exam #1, February 25, 2008

1. Find the following:

(a)

$$\int \sin^{100}(x)\cos^3(x)\,dx$$

(b)

$$\int \frac{3x^2-16x+5}{\sqrt{x^3-8x^2+5x+3}}\,dx$$

(c)

$$\int (x-\frac{3}{2})\sin(x^2-3x)\,dx$$

(d)

$$\int x\ln x^3\,dx$$

(e)

$$\int e^{\sqrt{x}}dx$$

(f)

$$\int_0^{\frac{\pi}{4}} \tan^5 x\sec^2 x\,dx$$

(g)

$$\int_2^4 f'(x)\sin(f(x))dx$$

if f is a differentiable function on the interval $[0,20]$ such that $f(0)=3$, $f(2)=1$, $f(4)=7$, and $f(20)=5$.

(h)

$$\int_1^3 (2x-8)e^{-x}\,dx$$

(i)

$$\int_{-\infty}^3 \frac{1}{1+x^2}\,dx$$

(j)

$$\int_1^5 \frac{x}{x^2-4}\,dx$$

(k)

$$\int_0^1 x^s\ln x\,dx$$

where s is a constant not equal to -1

(l) $\bar{f}$, the average value of $f(x) = x^2$ over the interval $[3, 8]$.

2. Consider the region R in the xy-plane where $x \geq 0$ bounded by the graphs $y = x^3$ and $y = x^5$.

   (a) Calculate the area of R.

   (b) Consider the solid S whose base is R and whose cross-sections perpendicular to the x-axis are equilateral triangles. Find the volume V of S.

3. Consider the region R in the xy-plane bounded by $y = 0$, and $y = 9 - x^2$. Find the centroid of R.

4. Find the arc lengths of the following:

   (a) The part of the curve $y = 2x^{\frac{3}{2}}$ between the points $(1, 2)$ and $(4, 16)$.

   (b) The part of the curve given parametrically by $x = 1 + 3t^2$ and $y = 4 + 2t^3$ between the points $(1, 4)$ and $(4, 6)$.

5. Consider the region Q in the xy-plane bounded by the graphs $y = x$ and $y = (x - 2)^2$. Find the volume of the solid obtained by revolving Q about the x-axis.

6. Consider the differential equation
$$\frac{dy}{dx} + 2y = e^x.$$

   (a) Verify that
$$y = Ce^{-2x} + \frac{1}{3}e^x$$
   is a solution for any constant C.

   (b) Find the solution which satisfies $y(0) = 8$.

7. Consider the differential equation
$$y^2 \frac{dy}{dx} + 3\cos x = 0$$

   (a) Find the general solution to the above equation.

   (b) Find the particular solution which satisfies $y(\pi) = 2$.

8. Find the orthogonal trajectories of the following family of curves:
$$y = \frac{k}{x}.$$