MATHEMATICS 566 A1
Geometric Methods in Mechanics
Spring Semester 2008

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Lectures: MWF 10-11 in MCS 148

Text: *Symmetry in Mechanics: A Gentle, Modern Introduction*, by Stephanie Frank Singer, Birkhäuser Boston; 1st Ed., March 1, 2001; ISBN-10: 0817641459; ISBN-13: 978-0817641450.

My Office Hours: TBA

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Content: The goal of this course is to give a pedestrian introduction to a mathematical framework for describing classical mechanics in a manifestly coordinate independent fashion. This is desirable as the fundamental laws of physics should admit a formulation which does not depend on any particular choice of coordinates. This mathematical framework is differential geometry, an area which uses calculus to describe higher dimensional versions of surfaces (called *manifolds*) in a coordinate independent way. Differential geometry plays a crucial role in a modern formulation of theoretical physics from classical mechanics to general relativity to quantum field theory.

In this course, we will focus primarily upon classical mechanical systems in the Hamiltonian formulation. Here, the time evolution of such a system corresponds to a trajectory in phase space. The differential geometric incarnation of phase space is called a *symplectic manifold* and trajectories on it are determined by a choice of a Hamiltonian function.

Another important idea in physics is that a conserved quantity typically arises from a continuous symmetry of the underlying physical system, e.g. momentum conservation is associated to translational invariance. In geometric terms, a continuous symmetry of a physical system corresponds to the action of a continuous symmetry group (called a *Lie group*) on the symplectic manifold which respects the Hamiltonian function. We will hence study Lie groups and their actions on symplectic manifolds.

We will also discuss constrained Hamiltonian systems and reduced phase space from the geometric point of view. If time permits, we will cover topics from general relativity and quantization.

Prerequisites: The material in the course is nontrivial so please make sure that you satisfy the prerequesites. The prerequisites to this course are multivariate calculus and some linear algebra. A knowledge of analysis or topology is useful but is not necessary. We will introduce these ideas as necessary.

Homework: Homework will be assigned periodically and will generally be due the following week. Late homework will not be accepted. Students may discuss homework with each other (and are encouraged to do so) but all written work must be prepared independently.

Exams: There will be a midterm exam and a final exam.

Grades: The average homework grade is worth 50% of the final grade, the midterm is worth 25%, and the final exam is worth 25%.