MATHEMATICS 727 A1
Introduction to Algebraic Topology
Fall Semester 2023

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Lectures: MWF 2:30-3:20 in CAS 316
Office Hours: TBA
Texts: This course will draw upon various texts as needed:

- *Algebraic Topology, A First Course*, by W. Fulton, ISBN-13: 978-0387943275
- *Algebraic Topology*, by A. Hatcher.
  - PDF file from Hatcher's web site.
  - And a list of corrections.

Content: Roughly speaking, algebraic topology seeks to study topological spaces by systematically associating to each topological space an algebraic object (say, a group or a vector space) in such a way that continuous maps between topological spaces induce natural maps between the corresponding algebraic objects (such as group homomorphisms or linear maps) which behave well under compositions of maps. By studying the resulting algebraic structures, one can often obtain valuable information about the topological space such as, for example whether two topological spaces can be homeomorphic.

In practice, algebraic topology turn out to be an extremely important tool not only in the study of topology but also, for example, in the areas of differential and complex geometry as well as mathematical physics and quantum field theory.

An important example is the fundamental group $\pi_1(X, x_0)$ of a topological space X with a basepoint x_0. While $\pi_1(X, x_0)$ easily generalizes to homotopy groups $\pi_n(X, x_0)$, the latter can be difficult to compute.

A more accessible example is the collection of abelian groups $H_\bullet(X)$, the homology groups of X, and its "dual" construction $H^\bullet(X)$, the cohomology groups of X. While the homology groups are trickier to define than the homotopy groups, they are often easier to compute. We will study different models of homology and cohomology theory including simplicial and singular homology. Homology/cohomology theory yields familiar numerical invariants such as the Euler characteristic for a large class of spaces. When the topological space happens to be a smooth manifold, its cohomology is closely related (and determines) its de Rham cohomology.

We will also cover relative homology, the Mayer-Vietoris sequence, the cup product in cohomology, universal coefficient theorem, cap products, and Poincaré duality. We will discuss de Rham cohomology and Čech cohomology, as time permits.

Prerequisites: We assume familiarity with basic notions from topology, e.g. topological spaces, compactness, connectedness, metric spaces and continuity, and from algebra, e.g. groups and rings.

Homework: Homework will be posted periodically on the class site. Students may discuss homework with each other (and are encouraged to do so) but all written work must be prepared independently.

Final Presentation: Instead of a written exam, students will give a short presentation on a topic related to the course at the end of the semester. The topic will be chosen in consultation with the prof.

Course Grade: The grade for the course will be based upon the formula:

$$\text{Course Grade} = \frac{1}{3}\,(\text{Final Presentation}) + \frac{2}{3}\,(\text{Homework Average})$$