MATHEMATICS 731 A1
An Introduction to Lie Groups and Lie Algebras
Spring Semester 2021

Instructor: Takashi Kimura
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Lectures: MWF 11:15-12:05

Recordings: All lectures will be recorded and posted on Blackboard at https://learn.bu.edu.

CampusWire: We will use CampusWire as a discussion forum for the class. Registered students will receive an invitation to join via their `@bu.edu` email address.

Required Text: *Representations of Compact Lie Groups*, by T. Bröcker and T. tom Dieck; Springer; ISBN 978-3642057250.

Recommended Text: *An Introduction to Lie Groups and Lie Algebras*, by A. Kirillov, Jr.; Cambridge University Press; ISBN-13 978-0521889698

Differential Geometry Reference: *Introduction to Smooth Manifolds*, by J. Lee; Springer; ISBN-13: 978-1441999818

Office Hours: All office hours will be held on Zoom. Time TBA.

Class Web Page: http://math.bu.edu/people/kimura/now/731

Content: Lie groups and Lie algebras form an elegant subject lying at the interface of geometry and algebra. It has many important applications to algebra, geometry, differential equations, and physics – indeed, to any system possessing a smooth family of symmetries. In this course, we will study such symmetry groups and their actions on vector spaces.

A Lie group is a smooth manifold which is also a group whose multiplication map is smooth. The primary source of Lie groups are groups of matrices under matrix multiplication e.g. the set of all $n \times n$ invertible matrices with $\mathbb{R}$ entries, $GL(n, \mathbb{R})$, is a Lie group.

Lie groups typically arise as smooth symmetries of spaces or of a system of equations. For example, the n-dimensional rotation group $SO(n)$ acts as symmetries of $\mathbb{R}^n$ via matrix multiplication. Such a linear action is a representations of the Lie group.

A Lie algebra is a vector space $\mathbf{g}$ with a skew symmetric bilinear map $\mathbf{g} \times \mathbf{g} \to \mathbf{g}$, $(X, Y) \mapsto [X, Y]$, satisfying the Jacobi identity, $[[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y] = 0$. The tangent space to the identity element of a Lie group is a Lie algebra. The Lie algebra, $so(n)$, associated to the Lie group $SO(n)$ may be interpreted as the infinitesimal rotations of $\mathbb{R}^n$.

We will study the relationship between Lie groups, Lie algebras, and their representations. We will cover Lie groups and Lie algebras, invariant vector fields, the exponential map, homogeneous spaces, representations, reducibility, character theory, induced representations, the representation ring, the maximal torus, Weyl groups, root systems, weight systems, Dynkin diagrams, classification of simple Lie algebras, and the Weyl character formula.

Prerequisites: A semester of differential topology (e.g. MA 721) at the level of Lee's *Introduction to Smooth Manifolds* will be useful.

Homework: Homework will be assigned periodically and it will be administered on Gradescope. Late homework will not be accepted. Students may discuss homework with each other (and are encouraged to do so) but all written work must be prepared independently.

Exams: The midterm exam is a written exam which will be administered on Gradescope. The final "exam" for the course will consist of giving a short presentation/talk on a topic related to the course.

Grades: Grades are based upon the formula:

$$\text{Final Grade} = \frac{1}{4}(\text{Midterm Exam}) + \frac{1}{2}(\text{ Homework Average}) + \frac{1}{4}(\text{Final Exam})$$