

# Contents

<b>1</b>	<b>Introduction and Overview</b>	1
1.1	Why Networks?	1
1.2	Examples of Networks	3
1.2.1	Technological Networks	3
1.2.2	Social Networks	5
1.2.3	Biological Networks	7
1.2.4	Information Networks	9
1.3	About this Book	11
<b>2</b>	<b>Preliminaries</b>	15
2.1	Background on Graphs	15
2.1.1	Basic Definitions and Concepts	16
2.1.2	Families of Graphs	18
2.1.3	Graphs and Matrix Algebra	20
2.1.4	Graph Data Structures and Algorithms	21
2.2	Background in Probability and Statistics	24
2.2.1	Probability	25
2.2.2	Principles of Statistical Inference	31
2.2.3	Methods of Statistical Inference: Tutorials	32
2.3	Statistical Analysis of Network Data: <i>Prelude</i>	42
2.4	Additional Related Topics and Reading	45
	Exercises	45
<b>3</b>	<b>Mapping Networks</b>	49
3.1	Introduction	49
3.2	Collecting Relational Network Data	50
3.2.1	Measurement of System Elements and Interactions	51
3.2.2	Enumerated, Partial, and Sampled Data	54
3.3	Constructing Network Graph Representations	56
3.4	Visualizing Network Graphs	58
3.4.1	Elements of Graph Visualization	58

3.4.2	Methods of Graph Visualization .....	60
3.5	Case Studies .....	63
3.5.1	Mapping ‘Science’ .....	65
3.5.2	Mapping the Internet .....	68
3.6	Mapping Dynamic Networks .....	74
3.7	Additional Related Topics and Reading .....	76
	Exercises .....	77
<b>4</b>	<b>Descriptive Analysis of Network Graph Characteristics .....</b>	<b>79</b>
4.1	Introduction .....	79
4.2	Vertex and Edge Characteristics .....	80
4.2.1	Degree .....	80
4.2.2	Centrality .....	88
4.3	Characterizing Network Cohesion .....	94
4.3.1	Local Density .....	94
4.3.2	Connectivity .....	97
4.3.3	Graph Partitioning .....	102
4.3.4	Assortativity and Mixing .....	111
4.4	Case Study: Analysis of an Epileptic Seizure .....	114
4.5	Characterizing Dynamic Network Graphs .....	116
4.6	Additional Related Topics and Reading .....	119
	Exercises .....	120
<b>5</b>	<b>Sampling and Estimation in Network Graphs .....</b>	<b>123</b>
5.1	Introduction .....	123
5.2	Background on Statistical Sampling Theory .....	126
5.2.1	Horvitz-Thompson Estimation for Totals .....	126
5.2.2	Estimation of Group Size .....	129
5.3	Common Network Graph Sampling Designs .....	131
5.3.1	Induced and Incident Subgraph Sampling .....	131
5.3.2	Star and Snowball Sampling .....	133
5.3.3	Link Tracing .....	136
5.4	Estimation of Totals in Network Graphs .....	137
5.4.1	Vertex Totals .....	137
5.4.2	Totals on Vertex Pairs .....	138
5.4.3	Totals of Higher Order .....	141
5.4.4	Effects of Design, Measurement, and Total .....	143
5.5	Estimation of Network Group Size .....	145
5.6	Other Network Graph Estimation Problems .....	149
5.7	Additional Related Topics and Reading .....	151
	Exercises .....	151

<b>6 Models for Network Graphs</b> .....	153
6.1 Introduction .....	153
6.2 Random Graph Models .....	154
6.2.1 Classical Random Graph Models .....	156
6.2.2 Generalized Random Graph Models .....	158
6.2.3 Simulating Random Graph Models .....	159
6.2.4 Statistical Application of Random Graph Models .....	162
6.3 Small-World Models .....	169
6.3.1 The Watts-Strogatz Model .....	169
6.3.2 Other Small-World Network Models .....	171
6.4 Network Growth Models .....	172
6.4.1 Preferential Attachment Models .....	173
6.4.2 Copying Models .....	176
6.4.3 Fitting Network Growth Models .....	178
6.5 Exponential Random Graph Models .....	180
6.5.1 Model Specification .....	180
6.5.2 Fitting Exponential Random Graph Models .....	185
6.5.3 Goodness-of-Fit and Model Degeneracy .....	187
6.5.4 Case Study: Modeling Collaboration Among Lawyers .....	188
6.6 Challenges in Modeling Network Graphs .....	191
6.7 Additional Related Topics and Reading .....	193
Exercises .....	195
<b>7 Network Topology Inference</b> .....	197
7.1 Introduction .....	197
7.2 Link Prediction .....	199
7.2.1 Informal Scoring Methods .....	201
7.2.2 Probabilistic Classification Methods .....	202
7.2.3 Case Study: Predicting Lawyer Collaboration .....	205
7.3 Inference of Association Networks .....	207
7.3.1 Correlation Networks .....	209
7.3.2 Partial Correlation Networks .....	212
7.3.3 Gaussian Graphical Model Networks .....	216
7.3.4 Case Study: Inferring Genetic Regulatory Interactions .....	220
7.4 Tomographic Network Topology Inference .....	223
7.4.1 Tomographic Inference of Tree Topologies .....	225
7.4.2 Methods Based on Hierarchical Clustering .....	228
7.4.3 Likelihood-based Methods .....	231
7.4.4 Summarizing Collections of Trees .....	234
7.4.5 Case Study: Computer Network Topology Identification .....	236
7.5 Additional Related Topics and Reading .....	241
Exercises .....	242

<b>8 Modeling and Prediction for Processes on Network Graphs .....</b>	245
8.1 Introduction .....	245
8.2 Nearest Neighbor Prediction .....	246
8.3 Markov Random Fields .....	249
8.3.1 Markov Random Field Models .....	249
8.3.2 Inference and Prediction for Markov Random Fields .....	252
8.3.3 Related Probabilistic Models .....	256
8.4 Kernel-based Regression .....	257
8.4.1 Kernel Regression on Graphs .....	258
8.4.2 Designing Kernels on Graphs .....	262
8.5 Case Study: Predicting Protein Function .....	266
8.6 Modeling and Prediction for Dynamic Processes .....	271
8.6.1 Epidemic Processes: An Illustration .....	272
8.6.2 Other Dynamic Processes .....	280
8.7 Additional Related Topics and Reading .....	281
Exercises .....	282
<b>9 Analysis of Network Flow Data .....</b>	285
9.1 Introduction .....	285
9.2 Gravity Models .....	287
9.2.1 Model Specification .....	288
9.2.2 Inference for Gravity Models .....	292
9.3 Traffic Matrix Estimation .....	297
9.3.1 Static Methods .....	298
9.3.2 Dynamic Methods .....	306
9.3.3 Case Study: Internet Traffic Matrix Estimation .....	310
9.4 Estimation of Network Flow Costs .....	316
9.4.1 Link Costs from End-to-end Measurements .....	317
9.4.2 Path Costs from End-to-end Measurements .....	321
9.5 Additional Related Topics and Reading .....	328
Exercises .....	330
<b>10 Graphical Models .....</b>	333
10.1 Introduction .....	333
10.2 Defining Graphical Models .....	334
10.2.1 Directed Graphical Models .....	335
10.2.2 Undirected Graphical Models .....	339
10.3 Inference for Graphical Models .....	342
10.4 Additional Related Topics and Reading .....	344
<b>Glossary of Notation .....</b>	345
<b>References .....</b>	347
<b>Author Index .....</b>	373
<b>Subject Index .....</b>	381