CAS MA539 – Methods of Scientific Computing

Boston University, Fall 2005

Homework 6: Stochastic Simulation.

(Due November 30)

1. RANDU: The RANDU pseudo-random number generator is defined through the following formula:

$$x_i = 65539 x_{i-1} \mathrm{mod} 2^{31} \ .$$

It was a common and trusted algorithm for quite some time, until it was found to have a particularly unsatisfactory lattice structure in $3D$.

   (a) Using standard rules of modular arithmetic, show that for this generator we can write

$$d_1 x_i + d_2 x_{i-1} + d_3 x_{i-2} = c 2^{31} \ ,$$

   for some constants (d_1, d_2, d_3) and c, and use the result of Marsaglia presented in class to determine the maximum number of hyperplanes in $(0,1]^3$ upon which pseudo-random numbers from this generator must sit.

   (b) Use two- and three-dimensional plots to illustrate the apparent "randomness" of sampled RANDU PRN's in pairs versus their clear lack of randomness in triplets.

2. Generate a Random Variable: You were shown a number of methods for the generation of random variables of a given distribution from pseudo-random numbers created by the computer. For this problem you are asked to create and implement an algorithm for generating independent samples of a random variable with a distribution of your choice. You may use the inverse method (for continuous or the modified version for discrete), or the acceptance rejection method. You should include in your write-up for this problem (i) definition of the random variable, (ii) description of the algorithm used, (iii) code for its implementation, and (iv) illustration as to the effectiveness of your method (e.g., comparison of histogram to density function, sample moments to population moments, etc.).

   NOTE: Please do *not* choose random variables shown in class, in particular the exponential or normal distributions.

3. Evaluate a Random Generator: It is good practice to be familiar with whatever pseudo-random number generator you might plan to use in simulation studies (now or in the future). For this third part of the assignment, you are asked to subject a production-caliber pseudo-random number generator (e.g., from Matlab, Splus, C, Fortran, etc.) to two tests, and to report the results in an organized fashion. These test should be:

   - Standard chi-square goodness-of-fit test for uniformity, as shown in class.
   - A test of your own creation. The general framework of chi-square testing was presented in class with the intention that that be available to you for this task, but you may implement other well-defined criteria (e.g., Kolmogorov-Smirnov, etc.) if you so wish.

   In writing up this section, you should explain briefly the characteristics of the generator that you tested (e.g., the source, the type (if known), relevant seed and other parameter settings, etc.). You should then summarize how the generator performed under your collection of tests. Finally, any specific details, such as tables, graphs, and code, should be included last.