Boston University
Summer I 2010

Number Theory
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Homework No.2

student:
due 05/24/2010

Problem A: Diophantine equations without solutions. (R)

P	C	N
-/6	-/2	-/2

1) Give a reason why the diophantine equation
$30x - 21y + 7 = 0$ has no integral solutions.

2) Give a reason why the diophantine equation
$x^2 + y^2 + z^2 = 47$ has no integral solutions.

3) Give a reason why the diophantine equation
$x^2 - 6x + y^2 + 12 = 0$ has no integral solutions.

Problem B: For the following two statements, first give a restatement, than give the contrary statement.
Example: If a natural number N can not be represented a sum of three squares, then the congruence $N \equiv 7 \pmod 8$ holds.
Equivalently, if a natural number is not congruent to 7 (mod 8), then the number could be represented as a sum of three squares.
Indeed, assume the contrary, that there exists a number $N \equiv 7 \pmod 8$, which could be written as $N = a^2 + b^2 + c^2$, for some $a, b, c \in \mathbb{Z}$.

Fact: Let $a, b \in \mathbb{N}$. The arithmetic progression $a, a+b, a+2b \ldots$ contains infinitely many prime numbers, if a and b are relatively prime.
Equivalently,...

Indeed, assume the contrary ...

Fact: If every proper divisor of an integer is even, then the integer must be a power of 2. Equivalently, ...

Indeed, assume the contrary...

Problem C: Find all solutions of the diophantine equation $6x + 10y = 4$, using any of the methods we learned in class.

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Problem D: Read Lockhart's article 'A Mathematician's Lament'. Write two paragraphs summarizing article's content and your position on the author views.

P	C	N
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