

Homework No.6

due 06/01/2009

Write proofs of the following four claims.

Use a single page for each problem.

Staple your work to this title page.

Remember to write neatly and to communicate clearly.

Problem A: Let m be a natural number. Prove that any set of $m^2 + 1$ integers, has a subset with $m + 1$ elements, such that the difference of any two numbers in the subset is divisible by m .

Problem B: Describe the set of primes which have a multiple that can be written with only 1's.

Problem C: Let p be a prime. A integer a is called *connected to p* if there are two consecutive integers, one of which is a perfect p -power, and the other is a multiple of a . For example, 9 is connected to the prime 5, since $125 = 5^3$ and $126 = 9 \times 14$.

Find all numbers connected to 7.

Problem D: Let $a, b, c, d \in \mathbb{N}$ be arbitrary. Prove that if $a|b$ and $c|d$, then $\gcd(a, c) | \gcd(b, d)$.