Boston University   Number Theory
Summer I 2009   Kalin Kostadinov

Homework No.7

student:
due 06/15/2009

Problem A: Sums of Squares:
1) Two of the following numbers are not sums of two squares.
Underline them: 123456789   123456   3145   266805.

2) Represent $2117 = 29 * 73$ as a sum of two squares, using the formula from
the lecture notes. How many essentially different representations are there?

Problem B: Go to Wikipedia and read the Zagier's 'one-sentence-proof' of
the two squares theorem. Then do the following:
1) Identify at least three general statements that are used in the proof and
write them explicitly and precisely in full details.
2) Find where exactly in the proof is used that $p \equiv 1 \pmod 4$

Problem C: Write a program that finds all four digit primes, which have the property that when read backwards they are also primes, i.e. in the two digit case 13 is prime, and so is 31.

Problem D: Let $m, n \in \mathbb{N}$. Prove that $\gcd(m - n, n) = \gcd(m, n)$. Is the analogical claim for the least common multiple true?