

## Homework No.9

due 06/22/2009

**Problem A:** Using Gauss Reciprocity Law, find out whether the two congruences have or have not solutions:

$$7x^2 + 34x + 17 \equiv 0 \pmod{97} \qquad 5x^2 + 6x + 25 \equiv 0 \pmod{83}$$

**Problem B:** The four consecutive integers 62, 63, 64, 65 are all composite, and the first is divisible by 2, the second by 3, the third by 4, and the fifth by 5. The five consecutive integers 122, 123, 124, 125, 126 have the analogous property - all composite, and divisible respectively by 2, 3, 4, 5, 6. Generalize this and show that for every natural number  $n$  there is a sequence with a similar property. Prove that in the sequence of natural numbers there are gaps of arbitrarily large size which contain no prime numbers.

Hint: you will need a *unary operation in postfix notation* in your answer.

**Problem C:** Write a computer program that takes as an input a natural number  $a$  and a prime number  $p$ , and outputs the correct one of the two sentences  
 $a$  is a **quadratic residue modulo  $p$**  or  
 $a$  is a **quadratic non-residue modulo  $p$**

**Problem D:** Write two paragraphs, the first one describing a couple of things in this course that you will remember, (e.g. I will always remember the derivation of Kepler's Laws about the planetary motion from Newton's Law of Universal Gravity. It is a great illustration of the power of multivariable calculus. I will also keep fond memories of the trivia games we played with the basic facts from the theory. They were a lot of fun and a great activity.) and the second paragraph addressing a future student in this course, in the vein of "I wish I was told/knew these, at the beginning of the course." (e.g. I wish someone told me that we will spend the second class in the library, I would have brought my ID so I could borrow some books. Also, it is a good idea to bring a stretcher for the breaks, exercising kept my mind fresh for the second half of the lectures. I am ashamed to mention it, but it was very stupid of me to start preparing the final project the night before the due date, you, dear future student should know better than this.) Send me your answers by e-mail.