Boston University Summer I 2010 Number Theory Kalin Kostadinov

Quiz I

student: 05/20/2010

Question 1: Diophantus lived duringin the city of Alexandria,in what is nowdaysBased on a surviving riddle about his life, helived to beyears old.

HintSThe end the Ice Age, around 10,000 BC; The Ancient World, around 250 AD; The Renaissance, around 1500 AD; Between WWI and WWII, around 1930; China;Egypt;France;Germany

...his boyhood lasted 1/6th of his life; he married after 1/7th more; his beard grew after 1/12th more, and his son was born 5 years later; the son lived to half his father's age, and the father died 4 years after the son. This is a riddle describing the life of Diophantus. How long was his life in years?

Question 2: There are diophantine equations with

- A) No solutions.
- B) Unique solution.
- C) Many solutions.
- D) All of the above.
- E) I don't know.

Question 3: Which of the diophantine equations is without solutions:

- A) 30x 21y + 7 = 0.
- B) $x^2 + y^2 + z^2 = 47$.
- C) $x^2 6x + y^2 + 12 = 0.$
- D) All of the above.
- E) I don't know.

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Question 4: Which of the following equations has (x, y) = (2, 3) amongst its solutions:

A) 3x + 5y - 17 = 0. B) $x^2 + y^2 = 12$. C) $y^2 = x^3 + 1$. D) None of the above. E) I don't know.

Question 5: What does $\sum_{k=0}^{9} 2^k = 1 + 2 + \dots + 512$ equals to? Answer:

Question 6: Circle the ones that CAN NOT be rational roots of the equation

 $50x^4 + 25x^3 - 43x^2 - 9x + 9$

Answer: $-2, \quad \frac{-5}{3}, \quad -1, \quad -\frac{3}{5}, \quad \frac{1}{2}, \quad \frac{3}{5}, \quad \frac{7}{2}, \quad 5$