

Quiz III

student:
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Question 1: For the following two statements, first give a restatement, than give the contrary statement.

Example: If a natural number N can not be represented a sum of three squares, then the congruence $N \equiv 7 \pmod{8}$ holds.

Equivalently, if a natural number is not congruent to $7 \pmod{8}$, then the number could be represented as a sum of three squares.

Indeed, assume the contrary, that there exists a number $N \equiv 7 \pmod{8}$, which could be written as $N = a^2 + b^2 + c^2$, for some $a, b, c \in \mathbb{Z}$.

Fact: Let $a, b \in \mathbb{N}$. The arithmetic progression $a, a + b, a + 2b \dots$ contains infinitely many prime numbers, if a and b are relatively prime.

Equivalently,...

Indeed, assume the contrary ...

Fact: If every proper divisor of an integer is even, then the integer must be a power of 2. Equivalently, ...

Indeed, assume the contrary...

Question 2: What is the result of running the following pseudo-code:

$N := 2009$ - - user input, a natural number greater than 1

Result := N

FOR I = 2 to SQUARE-ROOT(N) DO - - start a cycle

 IF I DIVIDES N

 THEN RESULT := I; GET-OUT-OF-CYCLE;

 ELSE SMILE;

NEXT I; - - end of cycle

PRINT(Result);

Your answer should be either a specific number (using input $N = 2009$,) or a single precise sentence of the type "The output of the program is the sum of the digits of the input"

Answer:

Question 3: Only one of the three Linear Diophantine equations has a solution. Use the Euclid algorithm to find which one, and then solve it.

A) $1173x + 527y = 1$.

B) $1173x + 527y = 11$.

C) $1173x + 527y = 17$.

Question 4: Factor 123123. You should get five different prime factors.

Question 5: Grandpa's age puzzle:

"My grandson is about as many days as my son is weeks, and my grandson is as many months as I am in years. My grandson, my son and I together are 100 years."

How old is Grandpa?