Problem # 1

$$f(x) = \int_0^{x^2} \sqrt{1+t^2}\,dt \qquad \text{find } f'(x)$$

Want to use FTC I, but have x^2 not x,
do u-sub

$u = x^2$
$du = 2x\,dx$

$$f(u) = \int_0^{u} \sqrt{1+t^2}\,dt$$ now it is in proper form for FTC I

using FTC I

$$\underbrace{\frac{df}{dx} = \frac{df}{du} \times \frac{du}{dx}}$$

$$\frac{df}{du} = \sqrt{1+u^2} \qquad \frac{du}{dx} = 2x$$

$$\frac{df}{dx} = 2x\sqrt{1+u^2}$$ ← plug in x value of u

Great job!

$$\frac{df}{dx} = 2x\sqrt{1+(x^2)^2}$$

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$$\boxed{\frac{df}{dx} = 2x\sqrt{1+x^4}}$$

① Find derivative $f'(x)$

$$f(x) = \int_0^{x^2} \sqrt{1+t^2}\, dt$$

- I can not use the first fundemental theorum of calculus here because x^2 is not proper, I have to use u substitution of the x^2 Great.

$$f(x) = \int_0^{u(x)} \sqrt{1+t^2}\, dt \qquad u(x) = x^2$$

- From here, I can used the derivative of $u(x) = x^2$ or $2x\,dx$ and use it in the chain rule.

$$\int_0^{u(x)} \sqrt{1+u^2} \cdot du \boxed{=} \sqrt{1+(x^2)^2} \cdot 2x = \boxed{2x\sqrt{1+x^4}} = f'(x)$$

Not equal

Good.

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① $F'(x) = f(x)$ Fundamental Theorem of Calculus

$$f(x) = \int_0^{x^2} \sqrt{1+t^2}\, dt$$

U Substitution

$u = x^2$

$du = 2x\, dx$ $\frac{du}{dx} = 2x$

$$f(x) = \int_0^{u} \sqrt{1+t^2}\, dt$$

$$f'(x) = \sqrt{1+(u)^2}\, du/dx$$

$$f'(x) = \sqrt{1+x^4} \cdot (2x)$$

$$\boxed{f'(x) = 2x\sqrt{1+x^4}}$$

Good.

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1. $f(x) = \int_0^{x^2} \sqrt{1+t^2}\, dt$

$u = x^2$

$du = 2x$

$f(x) = \left(\int_0^{u} \sqrt{1+t^2}\, dt\right) 2x$

$f'(x) = \sqrt{1+u^2} \cdot 2x$

$f'(x) = 2x\sqrt{1+(x^2)^2}$

$f'(x) = 2x\sqrt{1+x^4}$

To take the derivative of $f(x)$ normally the formula that would be used is $f'(t) = f(x)$, when given $f(x) = \int_a^x f(t)\,dt$. But, in this problem we do not have t, but a function of t and therefore must use the following formula to derive; if $f(x) = \int_a^{g(x)} f(t)\,dt$ then $f'(x) = f(g(x)) \cdot g'(x)$.

Great job!

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1) Derivative of

$$f(x) = \int_0^{x^2} \sqrt{1+t^2}\, dt$$

Due to the Fundamental Theorem of Calculus Part 1,

$$\frac{d}{dx}\left(\int_{g(h)}^{h(x)} f(t)\, dt \right) = f(h(t)) \cdot h'(t) - f(g(t)) \cdot g'(t)) \checkmark$$

$\therefore$

$$f'(t) = \left(\sqrt{1+(x^2)^2}\right)(2x - 0)$$

$$= 2x\sqrt{1+x^4}$$

Good job!

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1. $f(x) = \int_0^{x^2} \sqrt{1+t^2}\, dt = \int_0^{u} \sqrt{1+t^2}\, dt$

$u = x^2$

$du = 2x\,dx$

$\frac{du}{dx} = 2x$

u-substitution

$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}$

by Fundamental Theorem 1.

$= \sqrt{1+u^2} \cdot 2x = \sqrt{1+x^4} \cdot 2x$

$\boxed{2x\sqrt{1+x^4}}$

Great job! 16

1) $f(x) = \int_0^{x^2} \sqrt{1+t^2}\, dt$ $f'(x) = ?$

$g(u) = x^2$

$= \int_0^{g(x)} \sqrt{1+t^2}\, dt$

*This problem uses Fund. Thm of Calc VI to show $g(x) = \int_a^x f(t)dt = g'(\ldots$ $f(x)$ *

let $u = t$
$du = dt$

$= \int_0^{g(x)} \sqrt{1+u^2}\, du$

$(g(x))^2 = x^4$

$\left[\frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx} \right.$

$g(x) = x^2$
$g'(x) = 2x$

→ b/c of chain rule

$f'(x) = f'(g(x))\, g'(x)$

x^2 is a func… so need its c…

Good job.
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Deriv of $f(x) = \int \sqrt{1+u^2} = \sqrt{1+u^2}$

Deriv of $g(x) = 2x$.

$\therefore f'(x) = \sqrt{1+(x^2)^2} \cdot 2x$
$= 2x\sqrt{1+x^4}$

$= \boxed{2x\sqrt{1+x^4}}$